MA 241 Test 1 Version 2 Put all work and answers in the blue books. No Work=No Credit

1. (12 points) Evaluate $\int x\ln x\ dx$

2. (12 points) Evaluate $\int \frac{dx}{(x^2+9)^{\frac{3}{2}}}$ Hint:$x=3\tan\theta$

3. (12 points) Evaluate $\int_0^{\pi/2} \sin^2 x\ \cos^3 x\ dx$

4. (14 points) Find $\int \frac{x^3+3x^2+4x+8}{x^2(x^2+4)}\ dx$

5. (12 points) Determine whether the integral is convergent or divergent. Evaluate the integral if it is convergent. $\int_1^5 \frac{2}{(x-2)^3} dx$

6. (14 points) a) Sketch the region bounded by $y=\sqrt{x}$ and $x=3y$

   b) Set up (**DO NOT EVALUATE**) the integral needed to find the volume of the solid formed by revolving the region from part a) around the $x=-6$

7. (12 points) Sketch the region bounded by $y=x^2-2x$ and $y=x+4$ and then set up (**DO NOT EVALUATE**) the integral needed to find its area.

8. (12 points) Use Simpson's Rule and the given data to estimate the value of the integral $\int_0^{30} f(x)dx$

x	f(x)
0	6
5	11
10	3
15	5
20	-2
25	6
30	4

C2 T1 V2 Solutions

1. (12pts) $\int x \ln x \, dx$ LIATE

$u = \ln x$ $\quad v = \frac{1}{2}x^2$

$du = \frac{1}{x}dx$ $\quad dv = x\,dx$

$uv - \int v\,du$

$\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \frac{1}{x}\,dx$

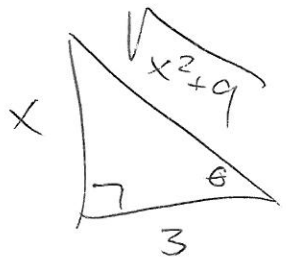
$\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\,dx$

$$\boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$$

2. (12pts) $x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$

$\frac{x}{3} = \tan\theta$



$$\int \frac{3\sec^2\theta\,d\theta}{(9\tan^2\theta + 9)^{3/2}}$$

$$= \int \frac{3\sec^2\theta\,d\theta}{(9\sec^2\theta)^{3/2}}$$

$$= \int \frac{3\sec^2\theta\,d\theta}{(3\sec\theta)^3}$$

$$= \int \frac{1}{3^3\sec\theta}\,d\theta$$

$$= \int \frac{1}{9}\cos\theta\,d\theta$$

$$= \frac{1}{9}\sin\theta + C$$

$$= \boxed{\frac{1}{9}\left(\frac{x}{\sqrt{x^2+9}}\right) + C}$$

3. (12pts)

$$\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx = \int_0^{\pi/2} \sin^2 x \cos^2 x \cos x \, dx$$

$$= \int_0^{\pi/2} \sin^2 x (1-\sin^2 x) \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int_0^1 u^2 (1-u^2) \, du$$

$$= \int_0^1 u^2 - u^4 \, du$$

$$= \frac{1}{3}u^3 - \frac{1}{5}u^5 \Big|_0^1$$

$$= \boxed{\frac{1}{3} - \frac{1}{5}}$$

4. (14pts) $\int \frac{x^3+3x^2+4x+8}{x^2(x^2+4)} dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} \, dx$

$$Ax(x^2+4) + B(x^2+4) + (Cx+D)x^2 = x^3+3x^2+4x+8$$

$$Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Dx^2 =$$

$A + C = 1 \quad C = 0$

$B + D = 3 \quad D = 1$

$4A = 4 \quad A = 1$

$4B = 8 \rightarrow B = 2$

$$\int \frac{1}{x} + \frac{2}{x^2} + \frac{1}{x^2+4} \, dx = \boxed{\ln|x| - \frac{2}{x} + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C}$$

5. (12pts)

$$\int_1^5 \frac{2}{(x-2)^3}\,dx$$

$$\int_1^2 \frac{2}{(x-2)^3}\,dx \qquad u = x-2,\quad du = dx$$

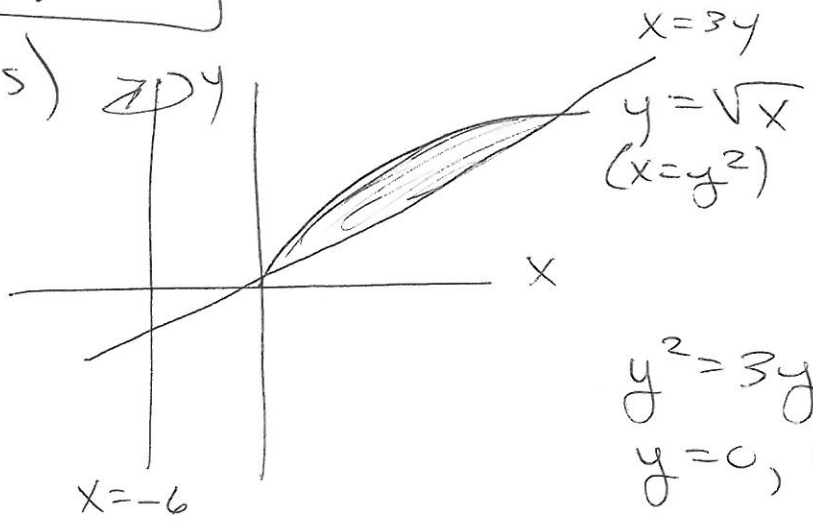
$$\int_{-1}^{0} \frac{2}{u^3}\,du = \lim_{t\to 0^-} \int_{-1}^{t} 2u^{-3}\,du$$

$$= \lim_{t\to 0^-} -u^{-2}\Big|_{-1}^{t}$$

$$= \lim_{t\to 0^-} -\frac{1}{t^2} + 1 \longrightarrow -\infty$$

divergent

6. (14pts)

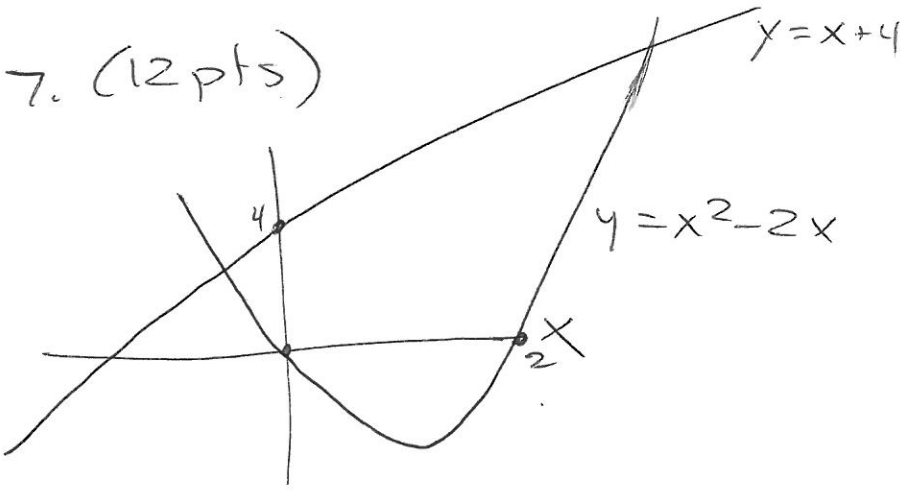


$$y^2 = 3y$$

$$y = 0,\ y = 3$$

$$V = \pi \int_0^3 (3y+6)^2 - (y^2+6)^2\,dy$$

7. (12 pts)



$x^2 - 2x = x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1)$

$$A = \int_{-1}^{4} x + 4 - (x^2 - 2x)\, dx$$

8. (12 pts)

$\Delta x = 5$

$$\int_0^{30} f(x)\,dx \approx \frac{5}{3}\left[6 + 4(11) + 2(3) + 4(5) + 2(-2) + 4(6) + 4\right]$$