MA 241 Test 1 Version 1 Put all work and answers in the blue books. No Work=No Credit

1. (12 points) Evaluate $\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$

2. (12 points) Evaluate $\int \frac{dx}{(x^2+4)^{\frac{3}{2}}}$ Hint:$x=2\tan\theta$

3. (12 points) Evaluate $\int x \ln x \, dx$

4. (14 points) Find $\int \frac{x^3+3x^2+4x+8}{x^2(x^2+4)} \, dx$

5. (12 points) Determine whether the integral is convergent or divergent. Evaluate the integral if it is convergent. $\int_3^6 \frac{2}{(x-4)^3} dx$

6. (14 points) a) Sketch the region bounded by $y=\sqrt{x}$ and $x=2y$

   b) Set up (**DO NOT EVALUATE**) the integral needed to find the volume of the solid formed by revolving the region from part a) around the $x=-3$

7. (12 points) Sketch the region bounded by $y=x^2-3x$ and $y=x+5$ and then set up (**DO NOT EVALUATE**) the integral needed to find its area.

8. (12 points) Use Simpson's Rule and the given data to estimate the value of the integral $\int_0^{24} f(x) dx$

x	f(x)
0	13
4	4
8	3
12	8
16	1
20	6
24	4

C2 T1 V1 Solutions

1. (12 pts) $\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$

$= \int_0^{\pi/2} \sin^2 x \cos^2 x \cos x \, dx$

$= \int_0^{\pi/2} \sin^2 x (1-\sin^2 x) \cos x \, dx$

$u = \sin x$

$du = \cos x$

$\int_0^1 u^2 (1-u^2) \, du$

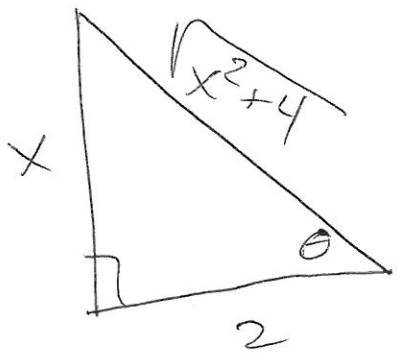
$= \int_0^1 u^2 - u^4 \, du = \frac{1}{3}u^3 - \frac{1}{5}u^5 \Big|_0^1 = \boxed{\frac{1}{3} - \frac{1}{5}}$

2. (12 pts) $x = 2\tan\theta$

$dx = 2\sec^2\theta \, d\theta$

$\int \frac{2\sec^2\theta \, d\theta}{(4\tan^2\theta + 4)^{3/2}} =$

$\frac{x}{2} = \tan\theta$



$\int \frac{2\sec^2\theta \, d\theta}{(4\sec^2\theta)^{3/2}}$

$= \int \frac{2\sec^2\theta \, d\theta}{(2\sec\theta)^3}$

$= \int \frac{1}{2\sec\theta} \, d\theta$

$= \int \frac{1}{2}\cos\theta \, d\theta$

$= \frac{1}{2}\sin\theta + C = \boxed{\frac{1}{2}\frac{x}{\sqrt{x^2+4}} + C}$

3. (12pts) $\int x \ln x \, dx$ LIATE

$u = \ln x \qquad v = \frac{1}{2}x^2$

$du = \frac{1}{x}dx \qquad dV = x\,dx$

$uv - \int v\,du$

$\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \frac{1}{x}\,dx$

$\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\,dx$

$\boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$

4. (14pts) $\int \frac{x^3+3x^2+4x+8}{x^2(x^2+4)}\,dx$

$= \int \frac{A}{x} + \frac{B}{x^2} + \frac{(Cx+D)}{x^2+4}\,dx$

$Ax(x^2+4) + B(x^2+4) + ((x+D)x^2 = x^3+3x^2+4x+8$

$Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Dx^2 =$

$A+C=1 \qquad C=0$

$B+D=3 \qquad D=1$

$4A=4 \rightarrow A=1$

$4B=8 \rightarrow B=2$

$\int \frac{1}{x} + \frac{2}{x^2} + \frac{1}{x^2+4}\,dx = \boxed{\ln|x| - \frac{2}{x} + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C}$

5. (12 pts)

$$\int_3^6 \frac{2}{(x-4)^3}\,dx$$

$$\int_3^4 \frac{2}{(x-4)^3}\,dx \qquad u = x-4, \quad du = dx$$

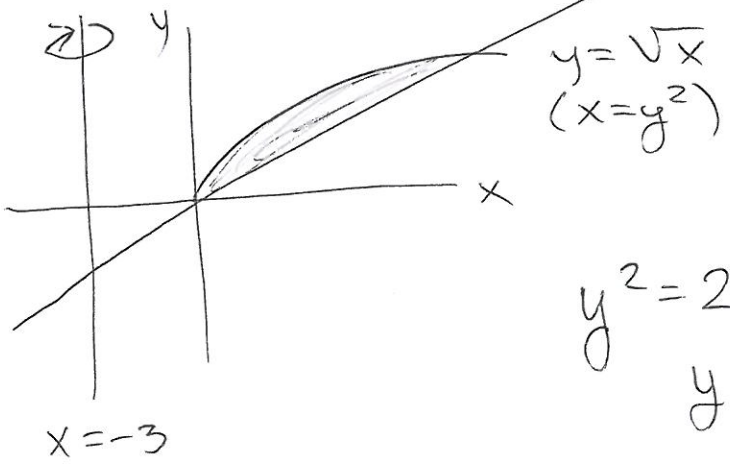
$$\int_{-1}^{0} \frac{2}{u^3}\,du = \lim_{t\to 0^-} \int_{-1}^{t} 2u^{-3}\,du$$

$$= \lim_{t\to 0^-} -u^{-2}\Big|_{-1}^{t} = -\frac{1}{t^2} + 1 \to -\infty$$

divergent

6. (14 pts)

a)

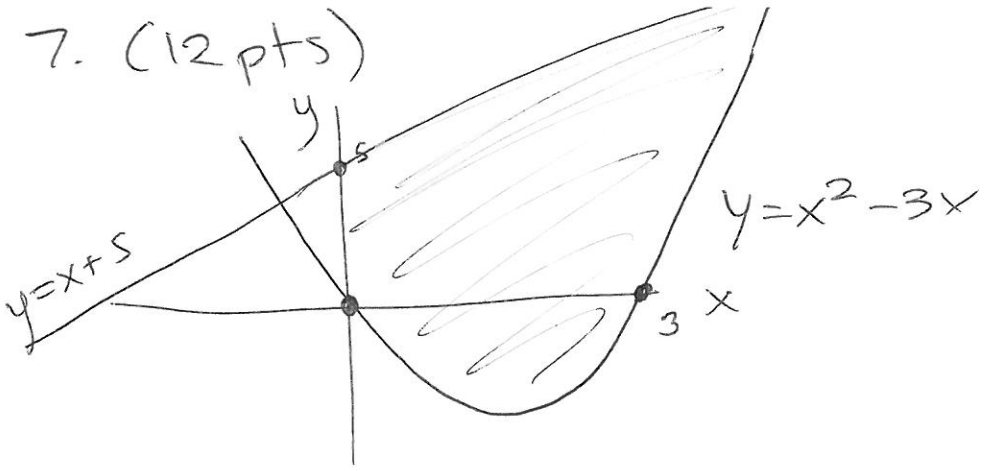


$$y^2 = 2y$$

$$y = 0, \; y = 2$$

b) $$V = \pi \int_0^2 [2y+3]^2 - [y^2+3]^2\,dy$$

7. (12 pts)



$$x^2-3x = x+5$$

$$x^2-4x-5 = 0$$

$$(x-5)(x+1) = 0$$

$$A = \int_{-1}^{5} x+5-(x^2-3x)\,dx$$

8. (12 pts)

$$\Delta x = 4$$

$$\int_0^{24} f(x)\,dx \approx \frac{4}{3}\left[13+4(4)+2(3)+4(8)+2(1)+4(6)+4\right]$$