Calculus II Honors Test 3 NO WORK=NO CREDIT!

1. (12 points) An isolated population is limited by its food supply to 4,000 individuals. The current population is 2,000 individuals; 40 years ago (t=0) it was 1,000 individuals.

a) Use the Logistic Equation to find a formula for P(t) the population at time t.

b) What is $\lim_{t\to\infty} P(t)$?

2. (17 points) Solve the Initial Value Problem $y''+2y'+10y=50e^{3x}$; $y(0)=3, y'(0)=-4$

3. (9 points) Find the general solution of $y''-4y'+4y=0$

4. (13 points) Use the differential equation $y''+y'-6y=f(x)$ along with the value of f(x) listed below to answer the following.

a) Find the complementary solution y_c

b) Find the form of y_p, but do **NOT** solve for the coefficients.

i) $f(x)=xe^{-x}$

ii) $f(x)=6\cos(3x)+5e^{2x}$

5. (15 points) A metal ball with a weight of 6 lb stretches a spring 3 inches. The damping constant is 2 and the spring is compressed 4 inches from its equilibrium and released with zero velocity. HINT : Gravity is 32 ft / s^2

a) If x(t) is the position of the metal ball at time t, formulate (**do NOT solve**) the IVP that describes the motion of the ball.

b) Determine what kind of damping this is and show your work. Do NOT multiply the numbers out!

6. (12 points) A cup of hot chocolate is 50°C when it is first made and 40°C 10 minutes later. The room it is sitting in is 30°C.

a) Find the formula for the temperature of the hot chocolate at time t. Hint: $T(t)=T_s+(T_0-T_s)e^{-kt}$

b) What is $\lim_{t\to\infty} T(t)$?

7. (7 points) Determine if the following sequences converge or diverge. If a sequence converges, find its limit. You do not need to show work.

a) $a_n=\dfrac{8^n}{3^{n+5}}$

b) $a_n=\dfrac{-19n^3+77n^2+2}{1-10n^2+3n^3}$

8. (5 points) What does it mean if a sequence is monotonic? (Keep your answers brief)

9.(10 points) Prove that if $\lim_{n\to\infty}|a_n|=0$, $\lim_{n\to\infty} a_n=0$. Fully justify your work.

C2H T3 Solutions

1. (12 pts)

a) $P(t) = \dfrac{C}{1+Ae^{-kt}} = \dfrac{4000}{1+Ae^{-kt}}$

$P(0) = 1000 = \dfrac{4000}{1+A}$ $\quad 1+A=4 \quad A=3$

$P(40) = 2000 = \dfrac{4000}{1+3e^{-40k}}$

$1+3e^{-40k} = 2$

$3e^{-40k} = 1$

$-40k = \ln\left(\frac{1}{3}\right)$

$-k = \frac{1}{40}\ln\left(\frac{1}{3}\right)$

$$P = \frac{4000}{1+3e^{\frac{1}{40}\ln\left(\frac{1}{3}\right)t}}$$

b) $\lim_{t\to\infty} P(t) = 4000$

2. (17pts) $y'' + 2y' + 10y = 50e^{3x}$

$r^2 + 2r + 10 = 0 \qquad r = \frac{-2 \pm \sqrt{4 - 40}}{2}$

$r = \frac{-2 \pm 6i}{2} = -1 \pm 3i$

$y_c = e^{-x}[C_1 \cos 3x + C_2 \sin 3x]$

$y_p = Ae^{3x} \qquad y_p' = 3Ae^{3x} \qquad y_p'' = 9Ae^{3x}$

$9Ae^{3x} + 2(3Ae^{3x}) + 10Ae^{3x} = 50e^{3x}$

$25A = 50$

$A = 2$

$y_p = 2e^{3x}$

$y = e^{-x}[C_1 \cos 3x + C_2 \sin 3x] + 2e^{3x}$

$y(0) = 3 = C_1 + 2 \qquad C_1 = 1$

$y = e^{-x}[\cos 3x + C_2 \sin 3x] + 2e^{3x}$

$y' = -e^{-x}[\cos 3x + C_2 \sin 3x] + e^{-x}[-3\sin 3x + 3C_2 \cos 3x] + 6e^{3x}$

$y'(0) = -1 + 3C_2 + 6 = -4$

$3C_2 = -9 \qquad C_2 = -3$

$$\boxed{y = e^{-x}[\cos 3x - 3\sin 3x] + 2e^{3x}}$$

3. (7 pts) $r^2 - 4r + 4 = 0$

$(r-2)^2 = 0$

$y = C_1 e^{2x} + C_2 x e^{2x}$

4. (13 pts)

a) $r^2 + r - 6 = 0$

$(r+3)(r-2) = 0$

$y_c = C_1 e^{-3x} + C_2 e^{2x}$

b) i) $y_p = (Ax + B)e^{-x}$

ii) $y_p = A\cos 3x + B\sin 3x + Cxe^{2x}$

5. (15 pts)

$F = mg$

$6 = m(32)$

$m = \frac{6}{32}$

$F = kx$ ← 3 inches

$6 = k\left(\frac{1}{4}\right)$

$k = 24$

a) $\frac{6}{32}x'' + 2x' + 24x = 0$ $\quad x(0) = -\frac{1}{3}$, $x'(0) = 0$

b) $b^2 - 4mk = 4 - 4\left(\frac{6}{32}\right)(24) < 0$

underdamping

6. (12pts) $T(t) = 30 + (50-30)e^{-kt}$

$= 30 + 20e^{-kt}$

$T(10) = 40 = 30 + 20e^{-10k}$

$\ln\left(\frac{1}{2}\right) = -10k$

$-k = \frac{1}{10}\ln\left(\frac{1}{2}\right)$

a) $\boxed{T(t) = 30 + 20e^{\frac{1}{10}\ln\left(\frac{1}{2}\right)t}}$

b) $\lim_{t\to\infty} T = 30$

7. (7pts)

a) $\lim_{n\to\infty} \frac{8^n}{3^{n+5}} \to \infty$ diverges

b) $\lim_{n\to\infty} a_n = \frac{-19}{3}$ converges

8. (5pts) It means it is either increasing or decreasing

9. (10pts) $\lim_{n\to\infty} |a_n| = 0$

$-|a_n| \le a_n \le |a_n|$

$\lim_{n\to\infty} -|a_n| = 0 = \lim_{n\to\infty} |a_n|$

So $\lim_{n\to\infty} a_n = 0$ by squeeze thm.