MA 241 Honors Test 2 NO WORK=NO CREDIT!!!

1. (12 points) Find the length of the curve represented by $x=1+6t^2$, $y=3+4t^3$, $0 \le t \le 1$

2. (8 points) Find the average value of the function $y=3x^2$ on $[1,3]$

3. (8 points) Write out the formulas needed (**DO NOT SOLVE THEM**) to find the centroid of the region bounded by $y=\ln x$, the x-axis, $x=1$, and $x=7$.

4. (11 points) A spring has a natural length of 7 inches. If a force of 4 lbs is required to stretch the spring from its natural length to 13 inches, find the work done stretching the spring from 13 inches to 19 inches. Include the appropriate units with your answer.

5. (10 points) Find the orthogonal trajectories of $y^3=kx^2$

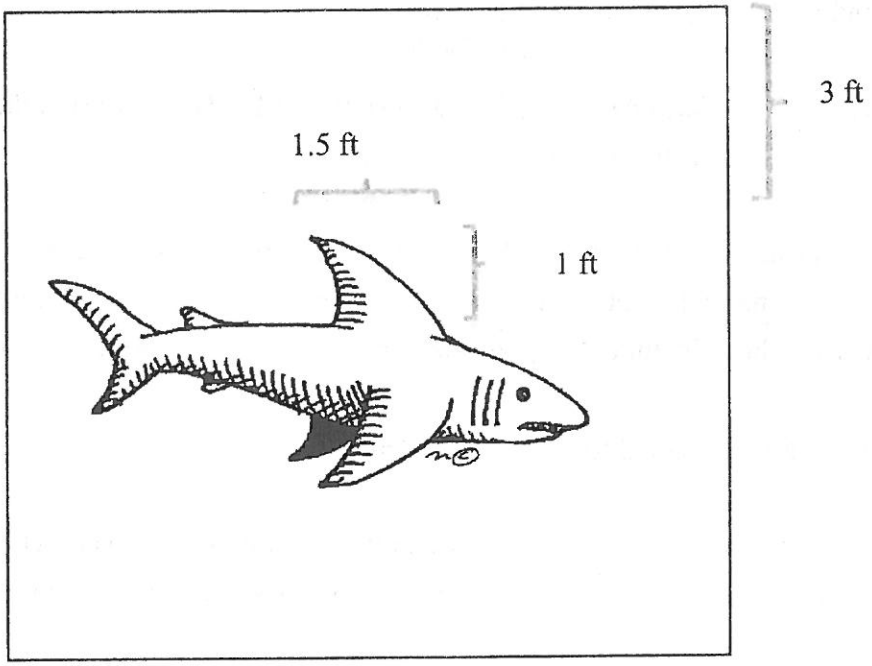
6. (10 points) Use Euler's method with a stepsize of 0.5 to estimate $y(0.5)$ and $y(1)$ for the Initial Value Problem (IVP) $y'=4x+y-1$, $y(0)=5$. Be sure to clearly indicate the values of your estimates.

7. (12 points) A trough 15m long is full of water. The ends of a trough are semicircles with radius 10m. Set up an integral (**DO NOT EVALUATE**) to find the work required to pump all of the water out of a spout located 2m above the top of the tank. The density of water is 1000 kg/m^3 and gravity is 9.8m/s^2.

8. (8 points) A tank contains 30 gallons of a solution consisting of 0.33lb of salt dissolved in water. A solution that contains 0.15lb of salt per gal enters the tank at a rate of 10gal/min. The solution is mixed and drains from the tank at the same rate. If y represents the amount of salt at time t, formulate the IVP (**DO NOT SOLVE IT**) to describe the rate of change of the amount of salt in the tank with respect to time.

9. (11 points) An integral equation is an equation containing an unknown function $y(x)$ and an integral involving $y(x)$. Solve the given integral equation. Hint: Use an initial condition obtained from the integral equation $y(x)=\int_0^x 2te^{-3y(t)}\,dt$

10. (10 points) Set up an integral (**DO NOT EVALUATE**) to find the hydrostatic force on one side of the dorsal fin of the shark shown on the back of the paper. Assume that the fin is triangular and has the given dimensions. The weight density of salt water is 64 lb/ft^3. Include the units with your answer.


3 ft
1.5 ft
1 ft

C2 HT2 Solutions

1. (12 pts) $\frac{dx}{dt} = 12t \quad \frac{dy}{dt} = 12t^2$

$$L = \int_0^1 \sqrt{(12t)^2 + (12t^2)^2}\, dt$$

$$= \int_0^1 \sqrt{144t^2 + 144t^4}\, dt$$

$$= \int_0^1 \sqrt{144t^2(1+t^2)}\, dt$$

$$= \int_0^1 12t\sqrt{1+t^2}\, dt \qquad u = 1+t^2$$

$$du = 2t\,dt$$

$$\int_1^2 6u^{1/2}\, du = 6\left(\frac{2}{3}\right)u^{3/2}\Big|_1^2$$

$$= \boxed{4\left(2^{3/2} - 1\right)}$$

2. (8 pts)

$$f_{aver.} = \frac{1}{3-1}\int_1^3 3x^2\, dx$$

$$= \frac{1}{2}\, x^3\Big|_1^3$$

$$= \frac{1}{2}(27 - 1) = \boxed{13}$$

3. (8pts)

$$(\bar{x},\bar{y}) = \left(\frac{\int_1^7 x \ln x \, dx}{\int_1^7 \ln x \, dx} , \frac{\int_1^7 \frac{1}{2}(\ln x)^2 \, dx}{\int_1^7 \ln x \, dx} \right)$$

4. (11pts) $F = kx$

$4\text{lb} = k(.5)$ ← 13 inches − 7 inches = 6 inches $= \frac{1}{2}$ ft

$k = 8$

$$W = \int_{1/2}^{1} 8x \, dx = 4x^2 \Big|_{\frac{1}{2}}^{1} = 4 - 4\left(\tfrac{1}{4}\right) = \boxed{3 \text{ ft-lbs}}$$

5. (10pts) $y^3 = kx^2$

$$3y^2 \frac{dy}{dx} = 2kx$$

$$\frac{dy}{dx} = \frac{2kx}{3y^2}$$

$$\boxed{y^2 + \frac{3}{2}x^2 = C}$$

$\perp$: $\frac{dy}{dx} = \frac{-3y^2}{2kx}$

k: y^3/x^2

$$\frac{dy}{dx} = \frac{-3y^2}{2\left(y^3/x^2\right)x} = \frac{-3x}{2y}$$

$$\int 2y \, dy = \int -3x \, dx \qquad y^2 = -\frac{3}{2}x^2 + C$$

6. (10pts) $x_0 = 0$ $\quad y_0 = 5$

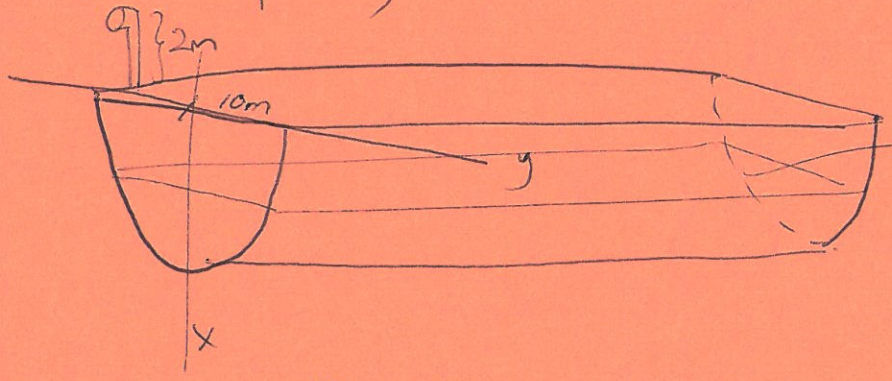
$x_1 = .5$

$x_2 = 1$

$$y(.5) \approx y_1 = y_0 + hf(x_0, y_0)$$
$$= 5 + \tfrac{1}{2}(4 \cdot 0 + 5 - 1)$$
$$= 7$$

$$y(1) \approx y_2 = y_1 + hf(x_1, y_1)$$
$$= 7 + \tfrac{1}{2}(4 \cdot \tfrac{1}{2} + 7 - 1)$$
$$= 7 + 4 = 11$$

7. (12pts)



$x^2 + y^2 = 100$

$A = 15 \cdot 2\sqrt{100 - x^2}$

$$W = \int_0^{10} 1000(9.8)\, 30\sqrt{100 - x^2}\,(x + 2)\, dx$$

8. (8pts)

$$\frac{dy}{dt} = (.15)(10) - 10\left(\frac{y}{30}\right)$$

$y(0) = .33$

9. (11pts) $\frac{dy}{dx} = 2xe^{-3y} = \frac{2x}{e^{3y}}$

$$\int e^{3y}\,dy = \int 2x\,dx$$

$$\frac{1}{3}e^{3y} = x^2 + C$$

$$y(0) = \int_0^0 2te^{-3y(t)}\,dt = 0$$

$$e^{3y} = 3x^2 + 3C$$

$$e^{3y} = 3x^2 + D$$

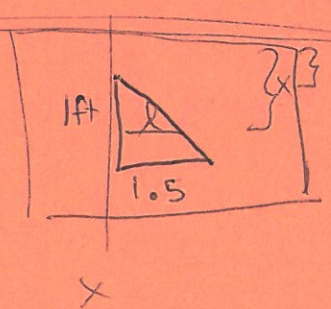
$$3y = \ln(3x^2 + D)$$

$$y = \frac{1}{3}\ln(3x^2 + D)$$

$y(0) = 0 \longrightarrow D = 1$

$$\boxed{y = \frac{1}{3}\ln(3x^2 + 1)}$$

10. (10pts)



$$\frac{l}{x-3} = \frac{1.5}{1}$$

$$l = 1.5(x-3)$$

$$\boxed{F = \int_3^4 64(1.5)(x-3)x\,dx \text{ lb}}$$