MA 241 Honors Test 2

1. (10 points) Find the length of the curve represented by $y = \frac{2}{3}(x^2+1)^{\frac{3}{2}}$, $0 \le x \le 1$

2. (10 points) Find the average value of the function $y = x\ln(x)$ on $[1,3]$
   Hint : Use Integration by Parts

3. (8 points)Write out the formulas needed to find the center of mass of the lamina bounded by $y = f(x)$, the x-axis, $x = a$, and $x = b$.

4. (10 points) A spring has a natural length of 100 cm. If it takes 12 Joules of work to stretch the spring from its natural length to 200 cm, find the work done stretching the spring from 200 cm to 250 cm. Include the appropriate units for your answer.

5. (10 points) Find the orthogonal trajectories of $y = kx$; graph both the trajectories and $y = kx$ for several values of k.

6. (10 points) Use Euler's method with a stepsize of 0.2 to estimate $y(1.2)$ and $y(1.4)$ for the initial value problem $y' = \sqrt{xy}$, $y(1) = 4$.

7. (10 points) A tank has the shape of an inverted circular cone with height 12 ft and base radius of 3 ft. The water level starts 2 ft down. Set up an integral (**DO NOT EVALUATE**) to find the work required to pump all of the water to a spout located 1 foot above the top of the tank. Use 62.5 lb/ft^3 for the weight density of water and include the appropriate units for the answer.

8. (12 points) A tank contains 1000L of a solution consisting of 100 kg of salt dissolved in water. A solution that contains 2 kg of salt per liter enters the tank at a rate of 5L/min. The solution is mixed and drains from the tank at the same rate. If y represents the amount of salt at time t, find an equation to represent the amount of salt in the tank after t minutes.

9. (10 points) The top of a vertical dam lies 11 m under the surface of the water. If the dam is a semicircle with radius 2 m. Set up an integral (**DO NOT EVALUATE**) to find the hydrostatic force against the gate. The density of water is 1000 kg/m^3 and gravity is $9.8 m/s^2$

10. (10 points) While carefully standing on the edge of a very tall cliff you see your Calculus text book attached to a 40 ft length of rope. Knowing that your book weighs about 5 lbs and the rope weighs 0.25 lb per linear foot, set up the necessary integral(s) (**DO NOT EVALUATE**) and equation(s) to find the total work done in lifting the book and the rope to the top of the cliff.

MA 241 Test 2 Honors

1. $$L=\int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

$$=\int_0^1 \sqrt{1+\left[(x^2+1)^{1/2}\cdot 2x\right]^2}\,dx$$

$$=\int_0^1 \sqrt{1+(x^2+1)(4x^2)}\,dx$$

$$=\int_0^1 \sqrt{1+4x^4+4x^2}\,dx$$

$$=\int_0^1 \sqrt{(1+2x^2)^2}\,dx$$

$$=\int_0^1 1+2x^2\,dx = x+\tfrac{2}{3}x^3\Big|_0^1 = \boxed{1+\tfrac{2}{3}}$$

2. $$F_{AVER}=\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$=\frac{1}{3-1}\int_1^3 x\ln x\,dx$$

LIATE

$u=\ln x \qquad V=\frac{1}{2}x^2$

$du=\frac{1}{x}dx \qquad dV=x\,dx$

$$=\frac{1}{2}\left[\frac{1}{2}x^2\ln x\Big|_1^3-\int_1^3 \frac{1}{2}x^2\,\frac{1}{x}\,dx\right]$$

$$=\frac{1}{2}\left[\frac{9}{2}\ln 3-\int_1^3 \frac{1}{2}x\,dx\right]$$

$$=\frac{1}{2}\left[\frac{9}{2}\ln 3-\frac{1}{4}x^2\Big|_1^3\right] = \boxed{\frac{1}{2}\left[\frac{9}{2}\ln 3-\frac{9}{4}+\frac{1}{4}\right]}$$

3. 8pts

$$\boxed{(\bar{x},\bar{y})=\left(\frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx},\ \frac{\int_a^b \frac{1}{2}[f(x)]^2\,dx}{\int_a^b f(x)\,dx}\right)}$$

4. $W = \int_0^1 kx\,dx = \frac{k}{2}x^2 \Big|_0^1 = \frac{k}{2} = 12$

$k = 24$

$W = \int_1^{1.5} 24x\,dx = 12x^2 \Big|_1^{1.5}$

$= \boxed{12(1.5)^2 - 12 \text{ J}}$

5. $y = kx \longrightarrow k = \frac{y}{x}$

$\frac{dy}{dx} = k$

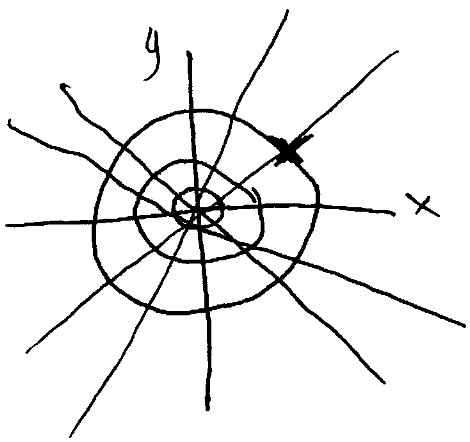
$\perp: \frac{dy}{dx} = -\frac{1}{k}$

$\frac{dy}{dx} = -\frac{x}{y}$

$\int y\,dx = \int x\,dx$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C$

$\boxed{\frac{1}{2}y^2 + \frac{1}{2}x^2 = C}$



6. $x_0 = 1$ $\quad y_0 = 4$

$x_1 = 1.2$ $\quad y(1.2) \approx y_1 = y_0 + hf(x_0, y_0)$

$x_2 = 1.4$

$= 4 + .2f(1,4)$

$= 4 + .2\sqrt{1(4)}$

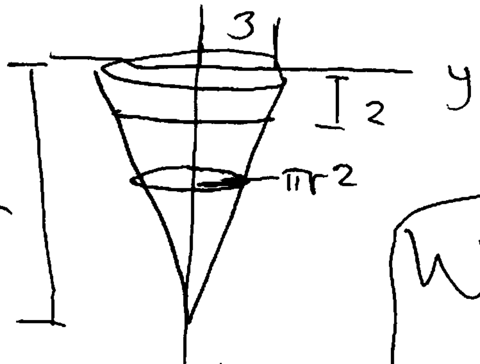
$= 4 + .4 = \boxed{4.4}$

$y(1.4) \approx y_2 = y_1 + hf(x_0, y_1)$

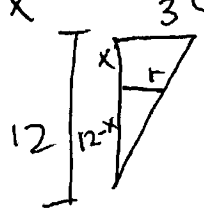
$= 4.4 + .2f(1.2, 4.4)$

$= \boxed{4.4 + .2\sqrt{(1.2)(4.4)}}$

7.



$$\boxed{W=\int_2^{12} 62.5\pi\left[\tfrac{1}{4}(12-x)\right]^2(x+1)\,dx \quad \text{ft-lb}}$$



$$\frac{3}{12}=\frac{r}{12-x}$$

$$\frac{1}{4}(12-x)=r$$

8. $y(0)=100$

$$\frac{dy}{dt}=\left(\frac{2}{2}\right)(5\ \text{L/m})-\left(\frac{y}{1000}\right)(5\ \text{L/min})$$

$$=10-\frac{y}{200}$$

$$=\frac{-1}{200}(y-2000)$$

$$\int\frac{dy}{y-2000}=\int-\frac{1}{200}dt$$

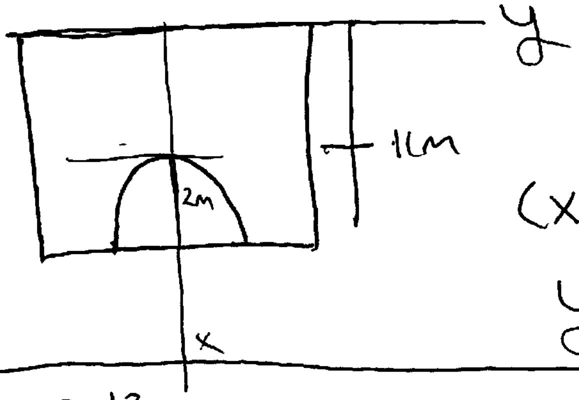
$$\ln|y-2000|=\frac{-1}{200}t+C$$

$$y-2000=Ke^{-\frac{t}{200}}$$

$$y=2000+Ke^{-\frac{t}{200}}$$

$$\boxed{y=2000-1900e^{-\frac{t}{200}}}$$

9.



$$(x-13)^2+y^2=4$$

$$y=\sqrt{4-(x-13)^2}$$

$$\boxed{F=\int_{11}^{13} 1000(9.8)\,2\sqrt{4-(x-13)^2}\;x\,dx}$$

10.

$$\boxed{W=5(40)+\int_0^{40} .25\,x\,dx}$$