MA 241 Honors Test 1 Put all work and answers in the blue books. No Work=No Credit

1. (10 points) Evaluate $\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$

2. (12 points) Evaluate $\int \frac{dx}{(x^2+4)^{\frac{3}{2}}}$ Hint:$x=2\tan\theta$

3. (10 points) Suppose that $f(1)=2$, $f(4)=7$, $f'(1)=10$, $f'(4)=3$, and f'' is continuous. Find the value of $\int_1^4 x f''(x)dx$

4. (14 points) Find $\int \frac{x^3+3x^2+4x+8}{x^2(x^2+4)} dx$

5. (12 points) Determine whether the integral is convergent or divergent. Evaluate the integral if it is convergent. $\int_3^6 \frac{2}{(x-4)^3} dx$

6. (12 points) a) Sketch the region bounded by $y=\sqrt{x}$ and $x=2y$
   b) Set up (**DO NOT EVALUATE**) the integral needed to find the volume of the solid formed by revolving the region from part a) around the $x=-3$

7. (12 points) Sketch the region bounded by $y=x^2-3x$ and $y=x+5$ and then set up (**DO NOT EVALUATE**) the integral needed to find its area.

8. (7 points) Set up (**DO NOT EVALUATE**) the integral needed to find the volume of the solid S, if the base of S is a circular disk with radius 3 and parallel cross-sections perpendicular to the base are squares.

9. (11 points) Use Simpson's Rule and the given data to estimate the value of the integral $\int_0^{24} f(x)dx$

x	f(x)
0	13
4	4
8	3
12	8
16	1
20	6
24	4

Calculus II Solutions

1. (10pts) $\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$

$= \int_0^{\pi/2} \sin^2 x \cos^2 x \cos x \, dx$

$\int_0^{\pi/2} \sin^2 x (1-\sin^2 x) \cos x \, dx$

$u = \sin x \qquad u(0) = 0$

$du = \cos x \, dx \qquad u(\pi/2) = 1$

$= \int_0^1 u^2 (1-u^2) \, du =$

$\int_0^1 u^2 - u^4 \, du = \frac{1}{3}u^3 - \frac{1}{5}u^5 \Big|_0^1$

$\boxed{\frac{1}{3} - \frac{1}{5}}$

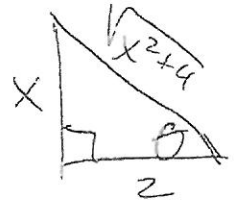
2. (12pts) $x = 2\tan\theta \longrightarrow \frac{x}{2} = \tan\theta$

$dx = 2\sec^2\theta \, d\theta$

$\int \frac{2\sec^2\theta \, d\theta}{(4\tan^2\theta + 4)^{3/2}} = \int \frac{2\sec^2\theta \, d\theta}{(4\sec^2\theta)^{3/2}}$

$= \int \frac{2\sec^2\theta \, d\theta}{8\sec^3\theta} = \int \frac{1}{4}\frac{1}{\sec\theta} \, d\theta$

$\int \frac{1}{4}\cos\theta \, d\theta = \frac{1}{4}\sin\theta + C$



$= \boxed{\frac{1}{4}\frac{x}{\sqrt{x^2+4}} + C}$

3. (10pts)

$$\int_1^4 x f''(x)\,dx$$

$u = x$ $\quad v = f'(x)$

$du = dx$ $\quad dv = f''(x)\,dx$

$$= x f'(x)\Big|_1^4 - \int_1^4 f'(x)\,dx$$

$$= x f'(x)\Big|_1^4 - f(x)\Big|_1^4$$

$$= 4f'(4) - 1f'(1) - f(4) + f(1)$$

$$4(3) - 10 - 7 + 2$$

$$= \boxed{-3}$$

4. (14pts)

$$\int \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}\,dx$$

$$A(x)(x^2+4) + B(x^2+4) + (Cx+D)x^2 = x^3 + 3x^2 + 4x + 8$$

$$Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Dx^2 =$$

$A + C = 1 \quad C = 0$

$B + D = 3 \quad D = 1$

$4A = 4 \quad A = 1$

$4B = 8 \rightarrow B = 2$

$$\int \frac{1}{x} + \frac{2}{x^2} + \frac{1}{x^2+4}\,dx = \boxed{\ln|x| - \frac{2}{x} + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C}$$

5. (12 pts)

$\int_3^4 \frac{2}{(x-4)^3}\,dx \qquad u = x-4,\ du = dx$

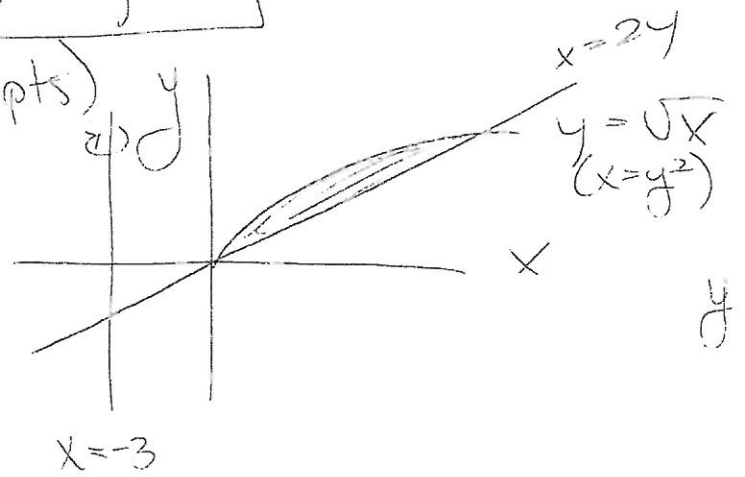
$\int_{-1}^{0} \frac{2}{u^3}\,du$

$= \lim_{t\to 0^-} \int_{-1}^{t} 2u^{-3}\,du$

$\lim_{t\to 0^-} -u^{-2}\Big|_{-1}^{t} = \lim_{t\to 0^-} -\frac{1}{t^2} + 1 \to -\infty$

divergent

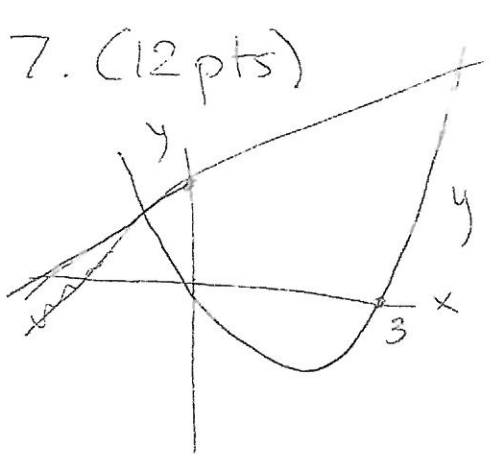
6. (12 pts)



$y^2 = 2y$

b) $V = \pi \int_0^2 \left[2y+3\right]^2 - \left[y^2+3\right]^2\,dy$

7. (12 pts)



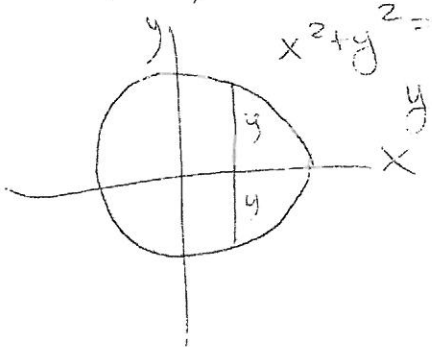
$x^2 - 3x = x + 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$A = \int_{-1}^{5} (x+5) - (x^2-3x)\,dx$

8. (7pts)



$x^2+y^2=9$

$y=\sqrt{9-x^2}$

$A=(2y)^2=4y^2$

$=4(9-x^2)$

$$V=\int_{-3}^{3} 4(9-x^2)\,dx$$

9. (11pts)

$$\int_0^{24} f(x)\,dx \approx \frac{4}{3}\left[13+4(4)+2(3)+4(8)+2(1)+4(6)+4\right]$$

$$=\frac{4}{3}\left[\underbrace{13+16}_{29}+\underbrace{6+32}_{39}+\underbrace{2+24}_{30}+4\right]$$