MA 241 Honors Test 1 Put all work and answers in the blue books. Only put 1 problem per page.

1. (12 points) Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx$

2. (12 points) Evaluate $\int \frac{1}{x^2\sqrt{x^2 - 81}} dx$ Hint: $x = 9\sec\theta$

3. (11 points) Approximate $\int_1^{17} \frac{1}{1 + y^4} dy$ using Simpson's rule with $n = 4$.

You do not need to simplify.

4. (13 points) Find $\int \frac{-x^2 + 4x - 5}{(x+1)(x^2+9)} dx$

5. (11 points) Determine whether $\int_4^{10} \frac{1}{x - 4} dx$ is convergent or divergent. If it converges, find its value.

6. (11 points) Sketch the region bounded by $y = 2 - x^2$ and $y = x$, and then find its area.

7. (20 points) a) Sketch the region bounded by $y = x^2$ and $x = y^2$

b) Set up (**DO NOT EVALUATE**) the integral to find the volume of the solid formed by revolving the region from part a) around the line $x = -3$

c) Set up (**DO NOT EVALUATE**) the integral to find the volume of the circular cone with height h and base radius r.

8. (10 points) Evaluate $\int \frac{\sec^2 x \tan^2 x}{\sqrt{16 - \tan^2 x}} dx$ using the tables listed below and write the number of the table you are using in your blue book.

T1. $\int \sqrt{a^2 - u^2} \, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$

T2. $\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1}\left(\frac{u}{a}\right) + C$

T3. $\int \frac{u^2}{\sqrt{a^2 - u^2}} du = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$

T4. $\int \frac{1}{u^2\sqrt{a^2 - u^2}} du = -\frac{1}{a^2 u}\sqrt{a^2 - u^2} + C$

MA 241 H Test 1 Solutions

1. $\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx = \int_0^{\pi/2} \sin^2 x \cos^2 x \cos x\,dx$

$= \int_0^{\pi/2} \sin^2 x(1-\sin^2 x)\cos x\,dx$ $\quad u=\sin x \quad u(0)=0$
$\quad du=\cos x \quad u(\pi/2)=1$

$= \int_0^1 u^2(1-u^2)\,du = \int_0^1 u^2-u^4\,du$

$= \frac{1}{3}u^3 - \frac{1}{5}u^5\Big|_0^1 = \boxed{\frac{1}{3}-\frac{1}{5}}$

2. $\int \frac{dx}{x^2\sqrt{x^2-81}}$ $\quad x = 9\sec\theta$
$\quad dx = 9\sec\theta\tan\theta\,d\theta$

$= \int \frac{9\sec\theta\tan\theta\,d\theta}{9^2\sec^2\theta\sqrt{81\sec^2\theta-81}} = \int \frac{9\sec\theta\tan\theta\,d\theta}{9^2\sec^2\theta\sqrt{81\tan^2\theta}}$

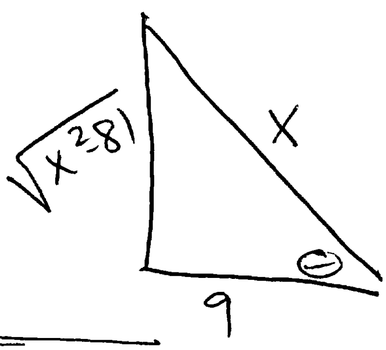
$= \int \frac{\cancel{9\sec\theta}\,\cancel{\tan\theta}\,d\theta}{9^2\sec^2\theta\,9\tan\theta} = \int \frac{1}{81}\frac{1}{\sec\theta}\,d\theta$

$= \int \frac{1}{81}\cos\theta\,d\theta = \frac{1}{81}\sin\theta + C$

$x = 9\sec\theta$

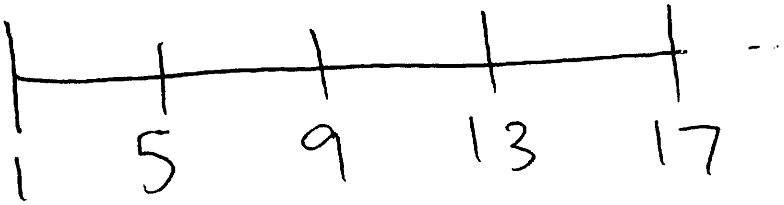
$\frac{x}{9} = \sec\theta$

$\frac{9}{x} = \cos\theta$



$= \boxed{\frac{1}{81}\frac{\sqrt{x^2-81}}{x} + C}$

3. $\Delta x = \frac{17-1}{4} = \frac{16}{4} = 4$ $\qquad f(y) = \frac{1}{1+y^4}$



$$S_4 = \frac{4}{3}\left[f(1) + 4f(5) + 2f(9) + 4f(13) + f(17)\right]$$

$$= \frac{4}{3}\left[\frac{1}{2} + \frac{4}{1+5^4} + \frac{2}{1+9^4} + \frac{4}{1+13^4} + \frac{1}{1+17^4}\right]$$

4. $\int \frac{-x^2+4x-5}{(x+1)(x^2+9)}\,dx = \int \frac{A}{x+1} + \frac{Bx+C}{x^2+9}\,dx$

$Ax^2 + 9A + Bx^2 + Bx + Cx + C = -x^2 + 4x - 5$

$$\left.\begin{array}{l} A+B=-1 \\ B+C=4 \end{array}\right\} A-C=-5 \quad \left.\begin{array}{l} \\ 9A+C=-5 \end{array}\right\}$$

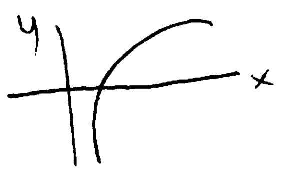
$10A = -10$
$A = -1$
$B = 0$
$C = 4$

$$\int \frac{-1}{x+1} + \frac{4}{x^2+9}\,dx = \boxed{-\ln|x+1| + \frac{4}{3}\tan^{-1}\left(\frac{x}{3}\right) + C}$$

5. $\int_4^{10} \frac{1}{x-4}\,dx = \lim_{t\to 4^+} \int_t^{10} \frac{1}{x-4}\,dx = \lim_{t\to 4^+} \ln|x-4| \Big|_t^{10}$

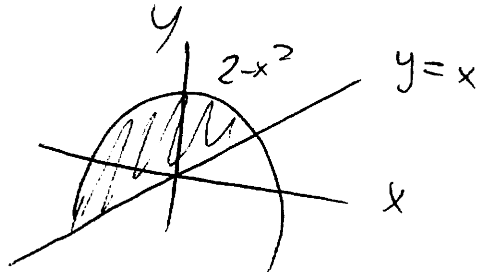
$= \lim_{t\to 4^+} \ln 6 - \ln|t-4| = \ln 6 - (-\text{"}\infty\text{"}) \to \infty$

Divergent



6.

$$A = \int_{-2}^{1} 2 - x^2 - x \, dx$$



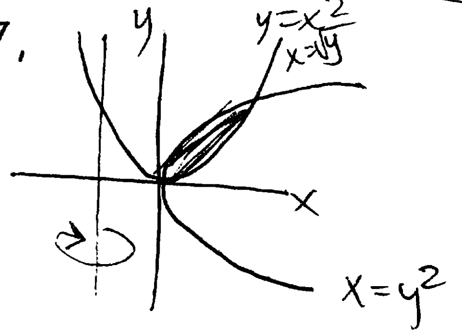
$$2 - x^2 = x$$
$$x^2 + x - 2 = 0$$
$$(x+2)(x-1) = 0$$

$$= 2x - \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 \Big|_{-2}^{1}$$

$$= 2 - \tfrac{1}{3} - \tfrac{1}{2} - \left[-4 + \tfrac{8}{3} - 2\right]$$

$$= 8 - \tfrac{9}{3} - \tfrac{1}{2} = \boxed{4.5}$$

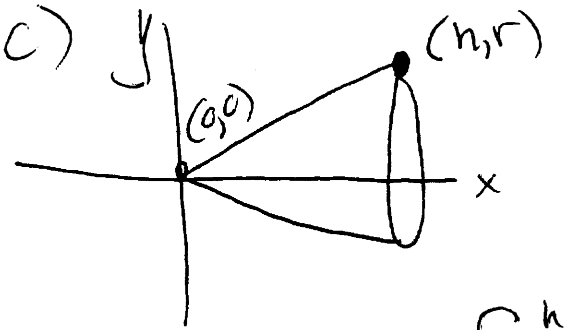
7.



b)

$$V = \pi \int_0^1 \left[3 + \sqrt{y}\right]^2 - \left[3 + y^2\right]^2 dy$$

c)



$$m = \frac{r-0}{h-0} = \frac{r}{h}$$

$$y = mx + b = \frac{r}{h}x$$

$$V = \int_0^h \pi \left[\frac{r}{h}x\right]^2 dx$$

8.

$$\int \frac{\sec^2 x \tan^2 x}{\sqrt{16 - \tan^2 x}} dx \qquad u = \tan x$$
$$du = \sec^2 x \, dx$$

$$= \int \frac{u^2}{\sqrt{16-u^2}} du = \boxed{-\frac{\tan x}{2}\sqrt{16 - \tan^2 x} + \frac{16}{2}\sin^{-1}\left(\frac{\tan x}{4}\right) + C}$$

$\boxed{T3}$