

## SOLUTIONS

SHOW ALL OF YOUR WORK!!!!

1. (10 points) Find the most general antiderivative of the function

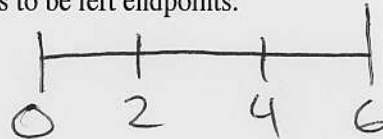
$$f(x) = 5x^{\frac{3}{7}} - \sec(x)\tan(x) + \frac{1}{x} - 12x^{-7} + \pi^3$$

$$F(x) = \frac{5x^{\frac{3}{7} + \frac{7}{7}}}{\frac{10}{7}} - \sec x + \ln|x| + 2x^{-6} + \pi^3 x + C$$

$$= \boxed{\frac{7}{2}x^{\frac{10}{7}} - \sec x + \ln|x| + 2x^{-6} + \pi^3 x + C}$$

2. (10 points) Evaluate the Riemann Sum of  $f(x) = x^3 + 4$  where  $0 \leq x \leq 6$  with three subintervals, taking the sample points to be left endpoints.

$$\Delta x = \frac{6-0}{3} = 2$$



$$L_3 = 2 (f(0) + f(2) + f(4))$$

$$= \boxed{2(4 + 2^3 + 4 + 4^3 + 4)}$$

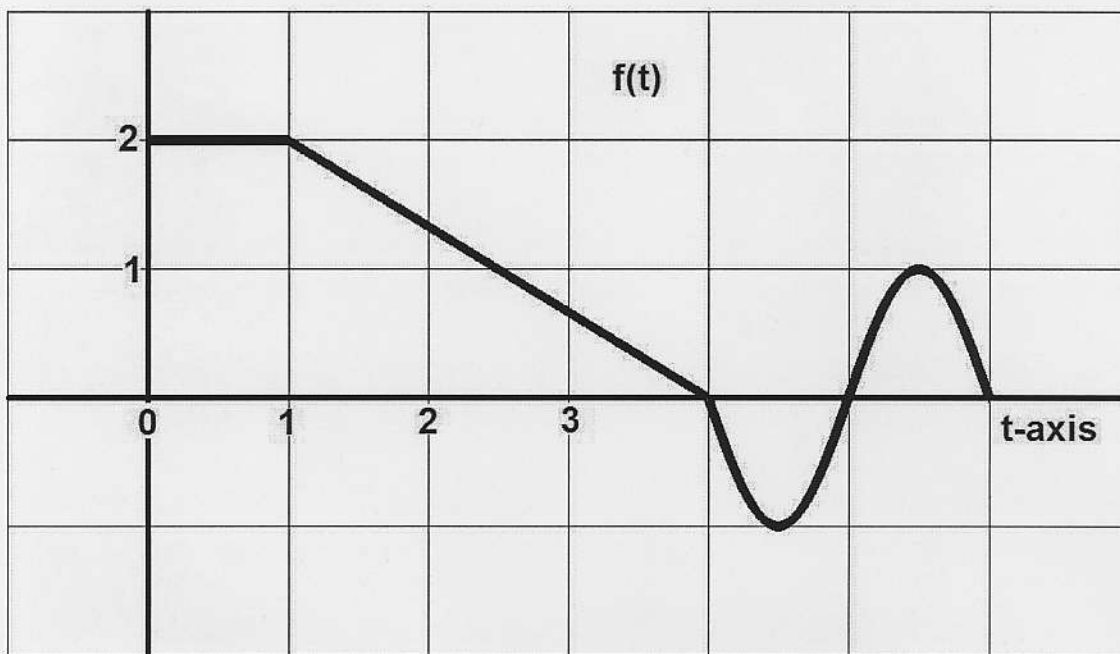
3. (10 points) a) If  $g(x) = \int_4^x \frac{\sec(t^9)(t-13t^2)}{\sin(t^2)} dt$ , find  $g'(x)$ .

$$g'(x) = \frac{\sec x^9 (x - 13x^2)}{\sin(x^2)}$$

b) What theorem did you use to find this?

Fundamental Theorem of  
Calculus

4. (20 points) If  $g(x) = \int_0^x f(t)dt$ , use the graph of  $f(t)$  below to answer the following questions.



a) Find  $g(0) = 0$

b) Find  $g(1) = 2$

c) Find  $g(4) = 2 + 3 = 5$

d) Find  $g'(x) = f(x)$

e) Find  $g'(0) = f(0) = 2$

f) Where is  $g$  increasing?  $(0, 4)$   $(5, 6)$

g) Where is  $g$  decreasing?  $(4, 5)$

h) Where does  $g$  have a local maximum?

i) Where does  $g$  have a local minimum?  $x = 4$

j) Find  $\int_4^6 f(t)dt = 0$   $x = 5$

5. (13 points) Evaluate the integral  $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$

$$= \int_1^0 \frac{-du}{1+u^2} = -\tan^{-1} u \Big|_1^0$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$u(0) = \cos 0 = 1$$

$$u\left(\frac{\pi}{2}\right) = 0$$

$$= -\tan^{-1} 0 + \tan^{-1} 1$$

$$= \boxed{\frac{\pi}{4}}$$

6. (13 points) Evaluate the integral  $\int_1^3 x \ln x dx$

LIATE

$$u = \ln x \quad v = \frac{1}{2}x^2$$

$$du = \frac{1}{x} dx \quad dv = x dx$$

$$\frac{1}{2}x^2 \ln x \Big|_1^3 - \int_1^3 \frac{1}{2}x^2 \frac{1}{x} dx$$

$$\frac{1}{2}x^2 \ln x \Big|_1^3 - \int_1^3 \frac{1}{2}x dx = \frac{9}{2} \ln 3 - \frac{1}{2}x^2 \Big|_1^3$$

$$= \frac{9}{2} \ln 3 - \frac{9}{4} + \frac{1}{4} = \frac{9}{2} \ln 2 - \frac{8}{4} = \boxed{\frac{9}{2} \ln 2 - 2}$$

7. (12 points) Evaluate the integral  $\int \cos^3 x \sin^5 x dx$

$$\int \cos^3 x \sin^5 x dx = \int \cos x \cos^2 x \sin^5 x dx$$

$$= \int \cos x (1 - \sin^2 x) \sin^5 x dx \quad u = \sin x \quad du = \cos x dx$$

$$= \int (1 - u^2) u^5 du = \int u^5 - u^7 du = \frac{1}{6} u^6 - \frac{1}{8} u^8 + C$$

$$= \boxed{\frac{1}{6} \sin^6 x - \frac{1}{8} \sin^8 x + C}$$

8. (12 points) Evaluate the integral  $\int x^2 e^{3x} dx$

Sign	$u$ & its derivatives	LIATE $dv$ & its antiderivatives
+	$x^2$	$e^{3x}$
-	$2x$	$\frac{1}{3}e^{3x}$
+	$2$	$\frac{1}{9}e^{3x}$
-	$0$	$\frac{1}{27}e^{3x}$

$$= \boxed{\frac{x^2}{3}e^{3x} - \frac{2x}{9}e^{3x} + \frac{2}{27}e^{3x} + C}$$

Extra Credit: (5 points) Using techniques developed in this class, solve the following problem.

A stone was dropped off a cliff and hit the ground with a velocity of  $-120$  ft/s.

What is the height of the cliff? Hint: Acceleration due to gravity  $= -32$  ft/s<sup>2</sup>

$$a = -32$$

$$v = -32t + C \quad (\text{stone dropped} \rightarrow C = 0)$$

$$v = -32t$$

$$p = -16t^2 + D$$

When the stone hits the ground position = 0  
and the velocity =  $-120$  ft/s so  
we can solve for  $t$   $-120 = -32t$  when  
the stone hits the ground  $\rightarrow t = \frac{15}{4}$

$$p\left(\frac{15}{4}\right) = -16\left(\frac{15}{4}\right)^2 + D = 0 = -225 + D = 0 \rightarrow$$

$$D = 225$$

Height of the cliff  $\boxed{225 \text{ ft}}$