MA 141 Test 3 - Version 2 No Work = No Credit!

1. (10 points) Write the formula for L(x), then find the linearization L(x) of f(x)=sin(x) at a=0 and use it to estimate sin(0.3)

2. (15 points) A spotlight on the ground shines on a wall 9 m away. If a man 2 m tall walks from the spotlight toward the wall at a speed of 1.1 m/s, how fast is the length of his shadow on the wall decreasing when he is 5 m from the building?

3. (25 points) Use $f(x)=\frac{x^2}{(x+3)}$ $f'(x)=\frac{x(x+6)}{(x+3)^2}$ $f''(x)=\frac{18}{(x+3)^3}$ to answer the following:

   a. State the domain of f(x)

   b. List all vertical asymptotes of f

   c. Find all critical numbers of f(x)

   d. Find the intervals where f(x) is increasing or decreasing.

   e. State the x-values of all maximums or minimums of f(x). Clearly label your answers.

   f. Find the intervals where f(x) is concave upwards or downwards

   g.List all inflection points

   h. Use your work from above to draw a rough graph of f. Label asymptotes and label the points where we have maxs/mins.

4. (12 points) Find the absolute maximum and minimum values of $f(x)=x^3-3x$ on [-3,0]

5. (23 points) Find the following limits. Verify L'Hospital's Rule applies before using it.

   a. $\lim_{x\to\frac{\pi}{2}} \frac{11-11\cos x}{\sin x}$

   b. $\lim_{x\to 0} \frac{e^{3x}-1-3x}{x^2}$

   c. $\lim_{x\to 0}(1+6x)^{\frac{1}{3x}}$

6. (15 points) $240 are available to fence in a rectangular garden. One side of the garden borders a river; here no fencing is used.On the other three sides it costs $2/ft. Find the dimensions of the largest possible garden.

C1 T3 V2 Solutions

1. (10 pts)

$L(x) = f(a) + f'(a)(x-a)$

$f(0) = 0$

$f' = \cos x$

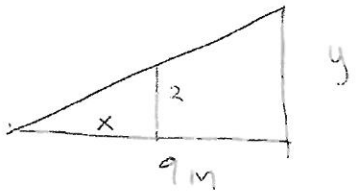
$f'(0) = 1$

$L(x) = 0 + 1(x-0)$

$= x$

$\sin(.3) \approx .3$

2. (15 pts)



$\frac{2}{x} = \frac{y}{9}$ $\quad \frac{18}{x} = y$

$y = 18x^{-1}$ $\quad \frac{dy}{dt} = -18x^{-2}\frac{dx}{dt}$

$\frac{dy}{dt} = -18(4)^{-2}(1.1) = -18\frac{1}{4^2}(1.1)$

$9 - x = 5$

$= -\frac{18}{16}(1.1)$

$\boxed{\frac{18}{16}}$ or $\frac{9}{8}$

3. (25 pts)

a) $x \neq -3$

b) $x = -3$

c) $x = 0,\ x = -6$

d)

Intervals	$f' = \frac{x(x+6)}{(x+3)^2}$	f
$(-\infty, -6)$	$f'(-7) > 0$	inc
$(-6, -3)$	$f'(-4) < 0$	dec
$(-3, 0)$	$f'(-1) < 0$	dec
$(0, \infty)$	$f'(1) > 0$	inc

e) $x = -6$ max

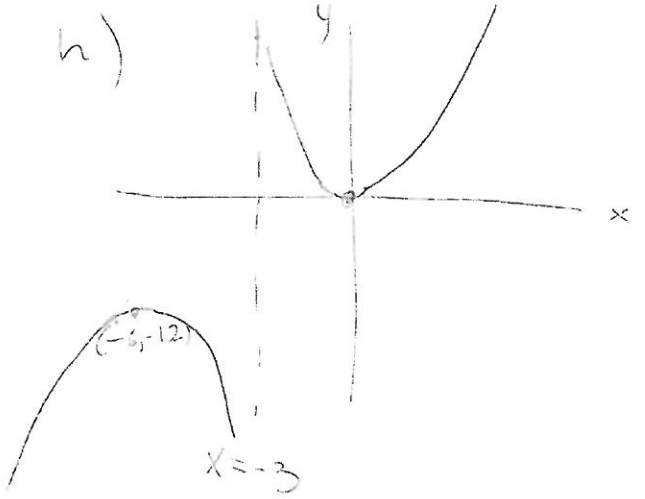
$x = 0$ min

f)

Intervals	$f'' = \frac{18}{(x+3)^3}$	f
$(-\infty, -3)$	$f''(-4) < 0$	cc dn
$(-3, \infty)$	$f''(0) > 0$	cc up

g) None

h)



$f(0) = 0$

$f(-6) = \frac{36}{-3} = -12$

4. (12 pts) $f' = 3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$ ~~$x = -1$~~ not in $[-3, 0]$

$f(-1) = -1 + 3 = 2$

$f(-3) = -27 + 9 = -18$

$f(0) = 0$

Abs max 2

Abs min -18

5 (23 pts)

a) $\dfrac{11 - 11\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \dfrac{11}{1} = \boxed{11}$

b) $\lim_{x\to 0} \dfrac{e^{3x} - 1 - 3x}{x^2}$ $\dfrac{e^0 - 1 - 0}{0} = \text{"}\frac{0}{0}\text{"}$

$= \lim_{x\to 0} \dfrac{3e^{3x} - 3}{2x}$ "$\frac{0}{0}$"

$= \lim_{x\to 0} \dfrac{9e^{3x}}{2} = \boxed{\dfrac{9}{2}}$

c) $\lim_{x\to 0} (1+6x)^{\frac{1}{3x}}$

$L = \lim_{x\to 0} (1+6x)^{\frac{1}{3x}}$

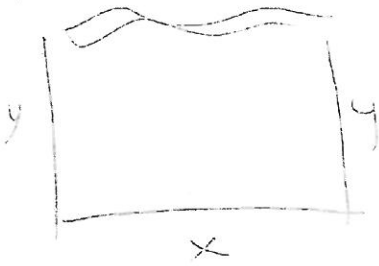
$\ln L = \lim_{x\to 0} \frac{1}{3x} \ln(1+6x)$

$= \lim_{x\to 0} \dfrac{\ln(1+6x)}{3x}$ "$\frac{0}{0}$"

$\ln L = \lim_{x\to 0} \dfrac{\frac{6}{1+6x}}{3} = \dfrac{6}{3} = 2$

$L = \boxed{e^2}$

6. (15pts)



$A = xy$

$240 = 2x + 2(2y)$

$120 = x + 2y$

$x = 120 - 2y$

$A = y(120 - 2y) = 120y - 2y^2$

$A' = 120 - 4y = 0$

$y = 30$

$x = 60$

$A'' = -4 < 0$ ∩ max ✓