Calculus I Test 2 Version 2 No work=No credit!

1. (12 points)
   a) State the definition of a derivative for a function f(x).
   b) Use the definition of a derivative to find $f'(x)$ for $f(x)=\dfrac{x}{6x+1}$

2. (12 points) To receive full credit, justify your answers as we have done in class.
   a) Find $\lim\limits_{x\to\infty}\dfrac{5e^x-2}{3-e^x}$
   b) What type of asymptote have you found in part a)?
   c) Find $\lim\limits_{x\to 1^-}\dfrac{-x}{x-1}$
   d) Find $\lim\limits_{x\to 1^+}\dfrac{-x}{x-1}$
   e) What type of asymptote is x=1?

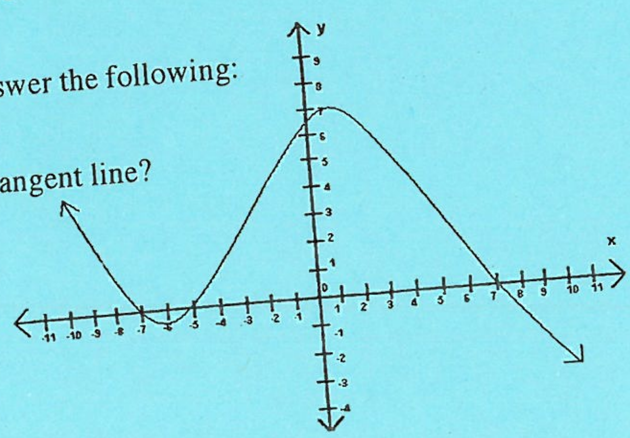
3. (47 points) Find the derivatives of the following functions
   a) $f(x)=\tan^{-1}(2x)+\log_4 x-e^3+6^x+\dfrac{1}{x^2}+\sqrt[3]{\ln(x)}+x^{\pi}$
   b) $y=\dfrac{2x-1}{x^3-7x}$
   c) $g(x)=e^{3x}\sec(x)$
   d) $y=\left(x^2-3x\right)^{\sqrt{x}}$

4. (10 points) Prove that $\dfrac{d}{dx}(\cot(x))=-\csc^2 x$

5. (10 points) Find the equation of the tangent line to $x^2+5xy+y^2=15$ at (2,1)

6. (3 points) There are 3 ways a function can fail to be differentiable at x=a. List 2 of them.

7. (6 points) Use the graph of f' given below to answer the following:
   a) On what intervals is f decreasing?
   b) At what values of x does f have a horizontal tangent line?



C1T2V2 Solutions

1. (12pts)

a) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

b) $f'(x) = \lim_{h\to 0} \dfrac{\dfrac{x+h}{6(x+h)+1} - \dfrac{x}{6x+1}}{h}$

$= \lim_{h\to 0} \dfrac{(x+h)(6x+1) - x(6x+6h+1)}{h(6x+6h+1)(6x+1)}$

$= \lim_{h\to 0} \dfrac{\cancel{6x^2} + \cancel{6x} + \cancel{6xh} + \cancel{h} - \cancel{6x^2} - \cancel{6xh} - x}{\cancel{h}(6x+6h+1)(6x+1)}$

$= \lim_{h\to 0} \dfrac{1}{(6x+6h+1)(6x+1)} = \dfrac{1}{(6x+1)^2}$

2. (12pts)

a) $\lim_{x\to\infty} \dfrac{5e^x - 2}{3 - e^x} \cdot \dfrac{\frac{1}{e^x}}{\frac{1}{e^x}} = \lim_{x\to\infty} \dfrac{5 - \frac{2}{e^x}}{\frac{3}{e^x} - 1} = -5$

b) horizontal

c) $\lim_{x\to 1^-} \dfrac{-x}{x-1} = \infty$

$\dfrac{-.9}{.9-1}$

d) $\lim_{x\to 1^+} \dfrac{-x}{x-1} = -\infty$

$\dfrac{-1.1}{1.1-1}$

e) vertical asymptote

a) $f'(x) = \frac{1}{1+(2x)^2}(2) + \frac{1}{\ln(4)\,x} + 0 + \ln(6)\,6^x - 2x^{-3} + \frac{1}{3}(\ln x)^{-\frac{2}{3}} + \pi x^{\pi-1}$

b) $$\boxed{y' = \frac{2(x^3-7x) - (2x-1)(3x^2-7)}{(x^3-7x)^2}}$$

c) $$\boxed{g'(x) = 3e^{3x}\sec x + e^{3x}\sec x\tan x}$$

d) $\ln y = \ln (x^2-3x)^{\sqrt{x}}$

$$\ln y = x^{1/2}\ln(x^2-3x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}x^{-1/2}\ln(x^2-3x) + x^{1/2}\left(\frac{1}{x^2-3x}\right)(2x-3)$$

$$\frac{dy}{dx} = y\left(\frac{1}{2}x^{-1/2}\ln(x^2-3x) + x^{1/2}\left(\frac{2x-3}{x^2-3x}\right)\right)$$

$$\boxed{\frac{dy}{dx} = (x^2-3x)^{\sqrt{x}}\left[\frac{1}{2}x^{-1/2}\ln(x^2-3x) + x^{1/2}\left(\frac{2x-3}{x^2-3x}\right)\right]}$$

4. (10pts)

$$\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\csc^2 x$$ ✓

5. (10pts) $2x + 5y + 5x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

(2,1) $4 + 5 + 10\frac{dy}{dx} + 2\frac{dy}{dx} = 0$

$$12\frac{dy}{dx} = -9$$

$$\frac{dy}{dx} = \frac{-9}{12} = \frac{-3}{4}$$

$$\boxed{y - 1 = \frac{-3}{4}(x-2)}$$

6. (3pts) discontinuous at a
vertical tangent at a
corner at a

7. (6pts)
a) (−7, −5) (7, ∞)
b) x = −7, x = −5, x = 7