Solutions

Test 2 141-002 Version 2

SHOW ALL OF YOUR WORK!!!!!! NO CALCULATORS!!

1.a (5 points) State the definition of a derivative for a function f(x).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

b. (10 points) Use the definition of a derivative to find f'(x) for $f(x) = \sqrt{3x+5}$.

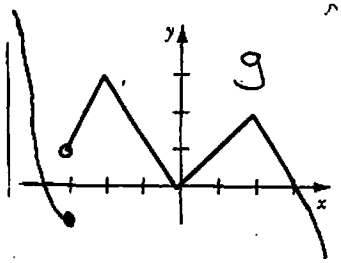
$$f'(x) = \lim_{h \to 0} \frac{\sqrt{3(x+h)+5} - \sqrt{3x+5}}{h} \cdot \frac{\sqrt{3(x+h)+5} + \sqrt{3x+5}}{\sqrt{3(x+h)+5} + \sqrt{3x+5}}$$

$$= \lim_{h \to 0} \frac{3(x+h)+5-(3x+5)}{h\left(\sqrt{3x+3h+5} + \sqrt{3x+5}\right)} = \lim_{h \to 0} \frac{3h}{h\left(\sqrt{3x+3h+5} + \sqrt{3x+5}\right)} = \frac{3}{2\sqrt{3x+5}}$$

2. (5 points) Refer to the graph of g(x) pictured below for the following questions:

a. Where is g discontinuous? $x = -3$

b. Where is g not differentiable? $x = -3 \quad x = -2 \quad x = 0, x = 2$



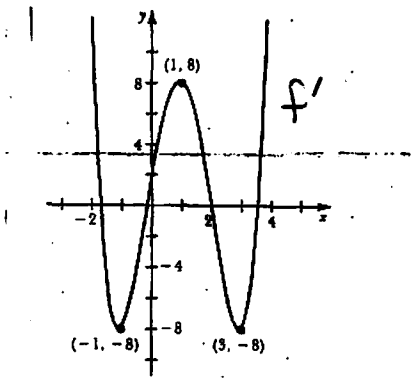
3. (10 points) Refer to the graph of f'(x) pictured below for the following questions:

a. On what intervals is f(x) increasing? $(-\infty, -1.6) \quad (-0.4, 2) \quad (3.5, \infty)$

b. Where does f have local maximums? $x = -1.6 \quad x = 2$

c. On what intervals is $f''(x) < 0$? $(-\infty, -1) \quad (1, 3)$

d. On what intervals is f(x) concave downwards? $(-\infty, -1) \quad (1, 3)$



4. (10 points) Find f'(x) if $f(x) = \cot(7x) + \frac{4}{x^2} + \sqrt[6]{x} + e^{\ln\sqrt{11}}$ $= \cot(7x) + 4x^{-2} + x^{1/6} + \sqrt{11}$

$f'(x) = -\csc^2(7x) \cdot 7 - 8x^{-3} + \frac{1}{6}x^{-5/6}$

5. (10 points) Find f'(x) if $f(x) = 2^x(x^3 + e^{-x})$

$f'(x) = 2^x \ln 2 \,(x^3 + e^{-x}) + 2^x(3x^2 - e^{-x})$

6. (10 points) Using the quotient rule find f'(x) if f(x) = tanx $= \frac{\sin x}{\cos x}$

$f'(x) = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x = 1}{\cos^2 x}$

$= \frac{1}{\cos^2 x} = \sec^2 x$

7. (15 points) Find an equation of the tangent line to $(x^2 + y^2)^2 = xy^2 + y$ at (0,1).

$2(x^2+y^2)\left(2x + 2y\frac{dy}{dx}\right) = y^2 + x2y\frac{dy}{dx} + \frac{dy}{dx}$

$2\left(2\frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$

$3\frac{dy}{dx} = 1 \quad \frac{dy}{dx} = 1/3 = m$

$y = \frac{1}{3}x + 1$

8. (10 points) Find $\frac{dy}{dx}$ where $y = (\sec x)^x$

$$\ln y = \ln(\sec x)^x$$

$$\ln y = x \ln \sec x$$

$$\frac{1}{y}\frac{dy}{dx} = \ln \sec x + x \cdot \frac{\sec x \tan x}{\sec x}$$

$$\frac{dy}{dx} = y(\ln \sec x + x \tan x)$$

9. (15 points) Refer to the chart for the following questions.

x	f(x)	f '(x)	g(x)	g '(x)
2	3	-2	5	4
3	1	14	2	6
5	7	11	9	13

a. If $F(x) = f(g(x))$, find $F'(3)$

$$F'(x) = f'(g(x)) \cdot g'(x)$$

$$F'(3) = f'(g(3))g'(3) = f'(2)\,6 = (-2)6$$

$$= -12$$

b. If $H(x) = \frac{f(x)}{g(x)}$, find $H'(2)$.

$$H'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

$$H'(2) = \frac{f'(2)g(2) - g'(2)f(2)}{[g(2)]^2}$$

$$= \frac{-2(5) - 4 \cdot 3}{(5)^2} = \frac{-22}{25}$$