P1

6.5 Applications to Physics & Engineering

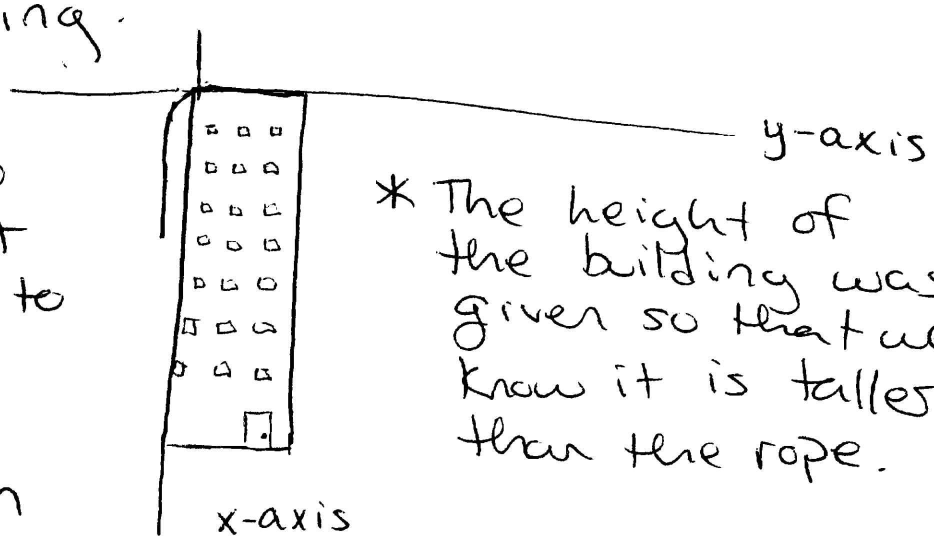
Work lifting...

From Calculus concepts & contexts 3rd edition by James Stewart:

$\boxed{\text{Ex1}}$ A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high.

a) How much work is done in pulling the rope to the top of the building?

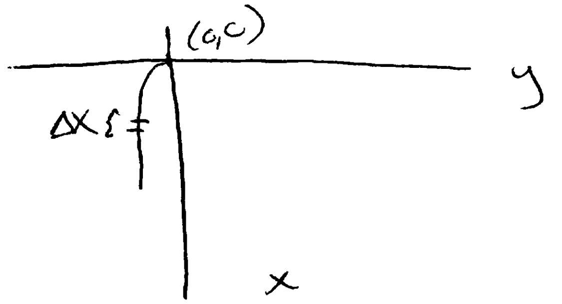
b) How much work is done in pulling half the rope to the top of the building?



* We'll need to use W=Fd, but since we need to lift the top of the rope 0 ft and the bottom 50 ft, we'll have to divide the rope up into small pieces and calculate the work to lift each piece.

* The height of the building was given so that we know it is taller than the rope.

P2



The work to lift a small piece $= Fd$

$$= \underbrace{(0.5\ \text{lb/ft})(\Delta x)}_{F}\ x_i$$

how far we lift the piece.

The total work of lifting the whole rope will be $W = \lim_{n\to\infty} \sum_{i=1}^{n} .5 x_i \Delta x$ or

$$W = \int_0^{50} .5x\,dx = \frac{1}{4}x^2\Big|_0^{50}$$

$$= \frac{2500}{4} = \boxed{625\ \text{ft-lb}}$$

b) This has a similar set up to part a) except that we need to think of this as 2 problems. The work required to lift 1/2 of the rope to the top of the bilding

$= \int_0^{25} .5x\,dx$ + the work required to lift the other half of the rope 25 ft $= \int_{25}^{50} .5(25)\,dx$

* Note: the 2nd portion of the rope is uniformly lifted 25 ft.

P3

$$W = \int_0^{25} .5x\,dx + \int_{25}^{50} .5(25)\,dx$$

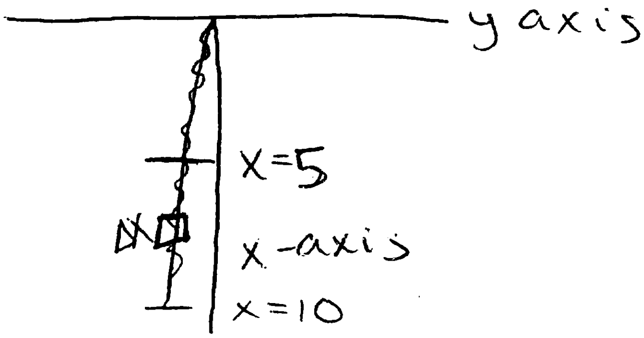
$$\frac{1}{4}x^2\Big|_0^{25} + \frac{1(25)}{2}x\Big|_{25}^{50}$$

$$\frac{1}{4}25^2 + \frac{1}{2}25^2$$

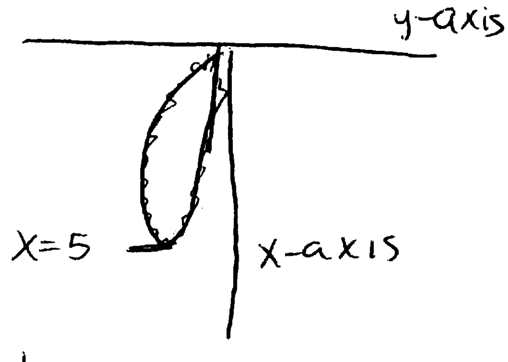
$$= \boxed{\frac{1875}{4}\text{ ft-lb}}$$

Ex2 A 10ft chain weighs 25 lb and hangs from the ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it's level with the upper end.

weight per unit length $= \frac{25\text{lb}}{10\text{ft}} = 2.5\text{ lb/ft}$

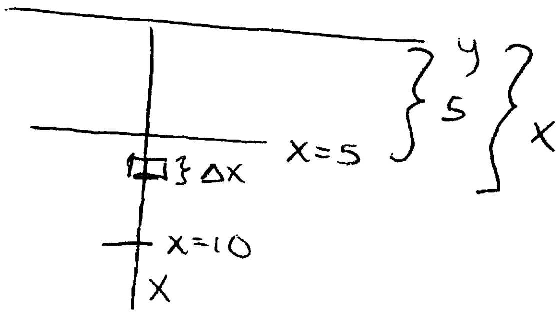


Situation



We want

This means that the bottom piece of chain is lifted 10ft while the middle to top part of the chain isn't lifted at all.



If we divide the chain into pieces & calculate the work necessary to move a piece we see that $d = 2(x-5)$. * Look at our piece if we just moved it $x-5$ its position would be at $x=5$. We need to go twice that.

$$W_{Piece} = Fd = \underbrace{\left(2.5 \frac{lb}{ft}\right)(\Delta x)}_{F} 2(x_i - 5)$$

or $$W_{total} = \int_5^{10} 2.5(2(x-5))\,dx$$

$$= \int_5^{10} 5x - 25\,dx$$

$$= \frac{5}{2}x^2 - 25x \Big|_5^{10}$$

$$= 250 - 250 - \left[\frac{125}{2} - 125\right]$$

$$= \boxed{125/2 \text{ ft-lb}}$$

EX 3 A bucket that weighs 4lb and a rope of negligible weight are used to draw water from a well that is 80ft deep. The bucket is filled with 40 lb of water and is pulled up at a rate of 2ft/s, but the water leaks out at a rate of .2lb/s. Find the work done in pulling the bucket to the top of the well.

* Total work done = work lifting the bucket + the work lifting the water.

Work lifting bucket = Fd = (4 lb)(80 ft)
= 320 ft-lb

* The bucket is pulled up at a rate of 2ft/s so after 40 seconds the bucket reaches the top of the well and if the water leaks out at a rate of .2 lb/s then 8 lbs of water is gone when the bucket reaches the top.

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The amount of water in the bucket is $\left(40 - \frac{x}{10}\right)$ after x feet. Note if we plug 80 in for x we get 32 lb as desired.

$$W_{water} = \int_0^{80} \underbrace{\left(40 - \frac{x}{10}\right)}_{\text{Force}} \cdot \underbrace{dx}_{\text{distance we move water}}$$

$$= 40x - \frac{x^2}{20} \Big|_0^{80} = 3200 - \frac{6400}{20} = 2880 \text{ ft-lb}$$

$$\text{Total work} = 320 + 2880 = \boxed{3200 \text{ ft-lb}}$$