6.5 Work Emptying a Tank

Recall: In class we derived the following formula $W = \int_a^b \rho\, A(x)\, x\, dx$

where a = where the liquid starts

b = bottom of liquid

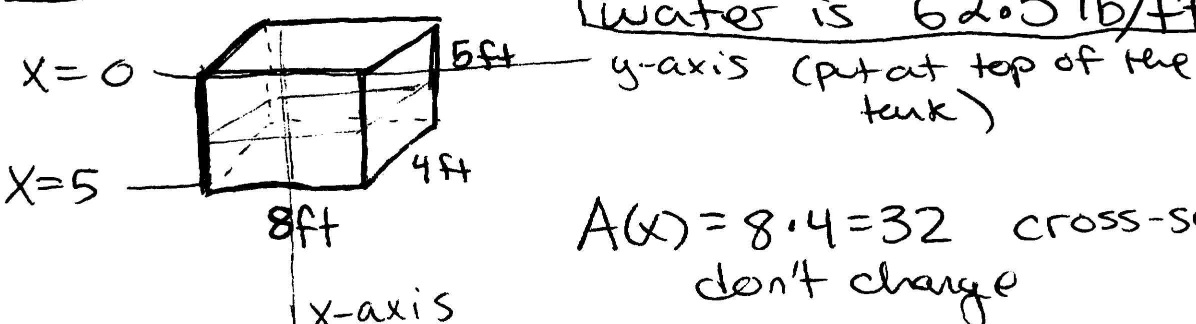
ρ = weight density of liquid (weight / volume)

$A(x)$ = area of a horizontal cross section

x = distance we are moving the liquid. If there is a spout this will need to be adjusted

Examples from Calculus by Rogawski

Ex1 Find the work required to pump all of the water to the top of the tank. Hint: the weight density of water is $62.5\ \text{lb}/\text{ft}^3$



$A(x) = 8 \cdot 4 = 32$ cross-sections don't change

$$\boxed{W = \int_0^5 62.5(32)\,x\,dx \ \text{ft-lb}}$$

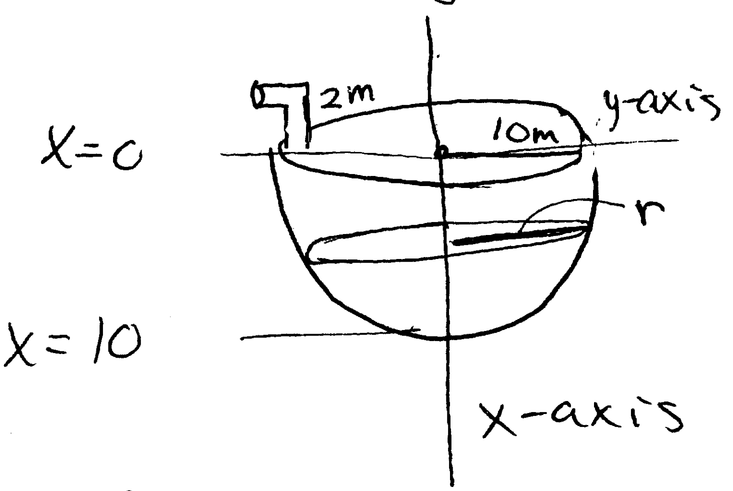
on your test I'll only ask you to set these up

b) What if the tank wasn't full & the water started 2 ft down.

* Everything is the same except the limits of integration. Remember a = where the water starts

$$\boxed{W = \int_2^5 62.5(32)\, x \, dx \quad \text{ft-lb}}$$

Ex 2 Find the work required to pump the water out of the spout shown. The density of water is 1000 kg/m³.
Hint: Gravity = 9.8 m/s²



* Start by putting the x-axis on a vertical line of symmetry

Cross-sections are circles, but they vary in size. We need to find the area of the circle as a function of x. Notice that the radius $r = y$ where y is the equation of the circle centered at $(0,0)$ with radius 10.

$$x^2 + y^2 = 10^2$$

$$r = y = \sqrt{100 - x^2} \qquad A(x) = \pi r^2 = \pi\left[100 - x^2\right]$$

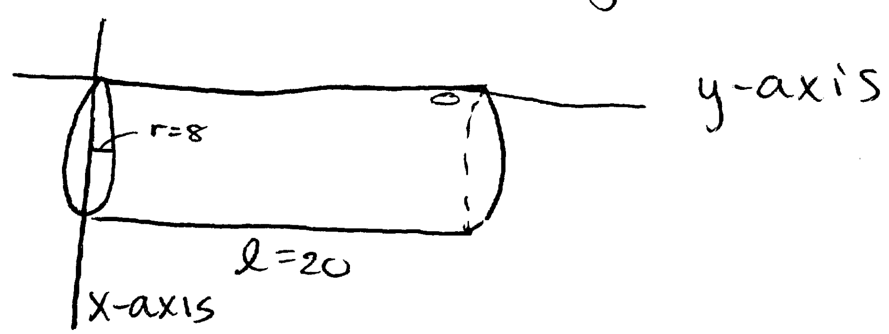
$$W = \int_0^{10} \underbrace{(1000\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})}_{\rho} \underbrace{\pi(100 - x^2)}_{A(x)} (x + 2)\, dx$$

ρ, remember mass × gravity = weight

2 m above the tank we need to go to get to the spout

The units here are joules.

$\boxed{\text{Ex 3}}$ Find the work required to pump all of the water out of a horizontal cylinder with radius of 8 m and length 20 m.

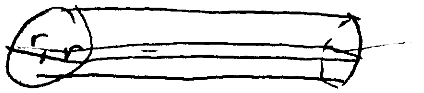


P4

We'll do this problem 2 ways. The 1st way will be like the set up for the other problems. We'll put the y-axis at the top of the tank.

$$W = \int_0^{16} \underbrace{(1000)(9.8)}_{\rho} A(x)\, x \, dx$$

Cross-sections are rectangles



$A = lw = 20w$

this is similar to the previous example $w = 2y$ where y is the circle with radius 8 centered at $(8,0)$

$$(x-h)^2 + (y-k)^2 = r^2$$

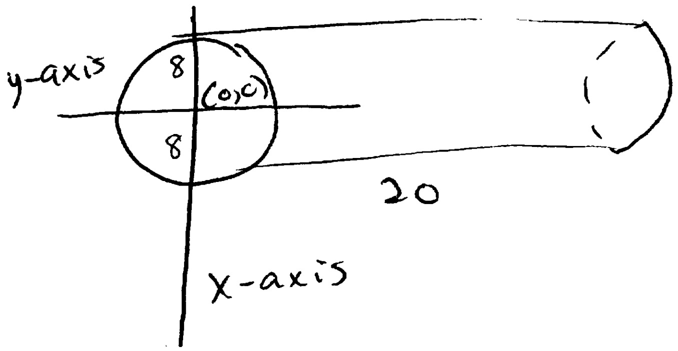
$$(x-8)^2 + y^2 = 64$$

$$y = \sqrt{64-(x-8)^2}$$

$$A(x) = 20\left(2\sqrt{64-(x-8)^2}\right)$$

$$\boxed{W = \int_0^{16} (1000)(9.8)\left(20\left(2\sqrt{64-(x-8)^2}\right)\right) x \, dx}$$

P5 The second way we can do this: put the y-axis so that it cuts the cylinder in half



The benefit of this is that now we have a circle centered at the origin $x^2+y^2=64$

$$y=\sqrt{64-x^2}$$

$$\boxed{W=\int_{-8}^{8} (1000)(9.8)\left(20\left(2\sqrt{64-x^2}\right)\right)(x+8)\,dx}$$