

# Self-dual and quasi self-dual algebras

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A *self-dual* algebra is an associative or Lie algebra  $A$  together with an  $A$  bimodule isomorphism  $A \cong A_{\text{op}}$ , where  $A_{\text{op}} = \text{Hom}_k(A; k)$ , the dual bimodule to  $A$  (considered as an  $A$  bimodule), and  $A_{\text{op}}$  is the same underlying  $k$  module as  $A_{\text{op}}$  but is an  $A$  bimodule whose left operation by an element  $a \in A$  is the same as the right operation by  $a$  on  $A_{\text{op}}$ , and similarly with left and right interchanged. This induces an isomorphism  $H^{\circ}E(A; A) \cong H^{\circ}E(A; A_{\text{op}})$ ; algebras with such an isomorphism are *quasi self-dual*. For these algebras  $H^{\circ}E(A; A)$  is a contravariant functor of  $A$ . They form a full subcategory of the category of the category of associative or Lie algebras, respectively. Finite dimensional associative self-dual algebras over a field are identical with symmetric Frobenius algebras (which are closely connected to 1+1 dimensional topological quantum field theory). Finite poset algebras are quasi self-dual.