

MA/OR/ST 706: Nonlinear Programming
Homework 4
Instructor: *Dr. Kartik Sivaramakrishnan*

INSTRUCTIONS

Due in class on Thursday March 20, 2008. All problems are from the 2nd edition of Nocedal and Wright unless otherwise specified. Please read Chapter 12 in Nocedal and Wright before beginning the assignment. No late homeworks will be accepted without prior instructor approval.

1. Problem 12.4, Page 352.
2. Problem 12.5, Page 352.
3. Problem 12.6, Page 352.
4. Problem 12.7, Page 352.
5. Problem 12.10, Page 352.
6. Problem 12.14, Page 353.
7. Problem 12.15, Page 353.
8. Problem 12.17, Page 353.
9. Problem 12.19, Page 354.
10. Consider the integer optimization problem

$$\begin{aligned} \min_x \quad & x^T C x \\ \text{s.t.} \quad & x_i^2 = 1, \quad i = 1, \dots, n \end{aligned} \tag{1}$$

where C is a symmetric matrix of size n (not assumed to be positive semidefinite) and x is a vector of size n . Note that the constraint $x_i^2 = 1$ implies that x_i is a binary variable that can only take the values -1 and 1 .

(a) Show that the Lagrangian dual of (1) is

$$\begin{aligned} \max_u \quad & \sum_{i=1}^n u_i \\ \text{s.t.} \quad & C - \text{Diag}(u) \succeq 0 \end{aligned} \tag{2}$$

where u is a vector of size n and $\text{Diag}(u)$ is a diagonal matrix of size n with the components of u along the main diagonal. The problem (2) is known as a semidefinite program and there are efficient algorithms to solve it to optimality.

- (b) Show that the optimal objective value to the semidefinite program (2) provides a lower bound on the optimal objective value of the integer program (1).