MA/OR/ST 706: Nonlinear Programming
Homework 2
Instructor: *Dr. Kartik Sivaramakrishnan*

INSTRUCTIONS

Due in class on Tuesday February 12, 2008. All problems are from the 2nd edition of Nocedal and Wright unless otherwise specified. Please read Chapter 3 in Nocedal and Wright before beginning the assignment. No late homeworks will be accepted without prior instructor approval.

1. Problem 3.1, Page 63. Both your codes should include the backtracking line search algorithm. You can use Kartik's MATLAB codes from the course webpage in your assignment. The local minimizer to the Rosenbrock function is $x^* = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Let x^k denote the approximation to the solution in the kth iteration. In the kth iteration of the steepest descent method, compute the ratio $\dfrac{||x^k - x^*||}{||x^{k-1} - x^*||}$, and in the kth iteration of Newton's method the ratio $\dfrac{||x^k - x^*||}{||x^{k-1} - x^*||^2}$. What do you observe?
2. Problem 3.2, Page 63.
3. Problem 3.3, Page 63.
4. Problem 3.5, Page 63.
5. Problem 3.6, Page 63.
6. Problem 3.7, Page 64.
7. Problem 3.8, Page 64.
8. Consider the function
$$f(x) \;=\; \frac{1}{2}(x_1^2 + \alpha x_2^2)$$
where $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\alpha \geq 1$. Suppose we use the method of steepest descent with exact line search starting from the point $x^0 = \begin{pmatrix} \alpha \\ 1 \end{pmatrix}$. Show that the method generates the sequence of points
$$x^k \;=\; \left(\frac{\alpha - 1}{\alpha + 1}\right)^k \begin{pmatrix} \alpha \\ (-1)^k \end{pmatrix}.$$

Hence, show that

$$\frac{f(x^{k+1})}{f(x^k)} \;=\; \left(\frac{\alpha-1}{\alpha+1}\right)^2.$$

which is the worst case bound in Theorem 3.3 on page 43.