MA/OR/IE 505-001: Linear Programming
Homework 4
Instructor: *Dr. Kartik Sivaramakrishnan*

INSTRUCTIONS

Due in class on Tuesday April 10, 2007. No late homeworks will be accepted without prior instructor approval. Please read the material in Chapter 26 of Chvátal; and the handout on branch and bound algorithms for integer programming (Chapter 7 in "Integer Programming" by Laurence Wolsey) on the course webpage before beginning the assignment.

1. Solve the block angular linear program (26.8) on page 435 of Chvátal (that we considered in class) using the Dantzig-Wolfe decomposition scheme. You can check your answers using Kartik's MATLAB Dantzig-Wolfe code posted on the course webpage.

2. Consider the following linear programming problem

$$\begin{array}{lrcl} \max & x_1 & & \\ \text{s.t.} & x_1 + 3x_2 & \leq & \frac{9}{4} \\ & 2x_1 - 3x_2 & \leq & 0 \\ & x & \in & X = \{(x_1, x_2) : \ 0 \leq x_1 \leq 1, \ 0 \leq x_2 \leq 1\}. \end{array}$$

   (a) Solve this problem graphically. Also, solve the problem directly with Kartik's revised simplex code from the course webpage. How many simplex iterations do you need?

   (b) We will now solve the problem using Dantzig-Wolfe decomposition scheme as discussed in class. Treat the first two inequalities as coupling constraints and the inequalities in X as the constraints in the subproblem. How many iterations do you need?

   (c) Does the reformulated master problem have multiple optimal solutions? Does the original problem have multiple optimal solutions? Explain you answers.

3. Consider a linear programming problem of the form

$$\begin{array}{lrcl} \max & c^T x + d^T y & & \\ \text{s.t.} & Ax + Dy & \leq & b \\ & Fx & \leq & f \\ & x, y & \geq & 0. \end{array}$$

(a) Suppose that we have access to a very fast subroutine for solving subproblems of the form

$$\begin{array}{lrcl} \max & h^T x & & \\ \text{s.t.} & Fx & \leq & f \\ & x & \geq & 0 \end{array}$$

for arbitrary cost vectors h. How would you go about decomposing the problem?

(b) Suppose that we have access to a very fast subroutine for solving subproblems of the form

$$\begin{array}{lrcl} \max & d^T y & & \\ \text{s.t.} & Dy & \leq & h \\ & y & \geq & 0 \end{array}$$

for arbitrary right hand side vectors h. How would you go about decomposing the problem?

4. Consider the two-variable integer program

$$\begin{array}{lrcl} \max & 9x_1 + 5x_2 & & \\ \text{s.t.} & 4x_1 + 9x_2 & \leq & 35 \\ & x_1 & \leq & 6 \\ & x_1 - 3x_2 & \geq & 1 \\ & 3x_1 + 2x_2 & \leq & 19 \\ & x_1 & \geq & 0 \text{ and integer} \\ & x_2 & \geq & 0 \text{ and integer.} \end{array}$$

Solve by branch and bound graphically and algebraically.

5. Consider the binary integer program

$$\begin{array}{lrcl} \max & -x_{n+1} & & \\ \text{s.t.} & 2x_1 + 2x_2 + \ldots + 2x_n + x_{n+1} & = & n \\ & x_i & \in & \{0,1\}, \;\; i = 1, \ldots, n+1. \end{array}$$

Show that any branch and bound algorithm that uses linear programming relaxations to compute upper bounds and branches by setting a fractional variable to either zero or one will require the enumeration of an exponential number of nodes when n is odd.