ALTERNATE APPROACH USING THE PRIMAL/DUAL SIMPLEX code

(a) Solve the original LP (1)

$$A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ 16 \end{bmatrix}, \quad C = \begin{bmatrix} 5 \\ 1 \\ -12 \\ 0 \end{bmatrix}$$

$eps = 1e-6$ and $B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

with Kaihk's revised simplex code

The optimal solution is

$$x^* = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \text{ and } y^* = \begin{bmatrix} -10 \\ 7 \end{bmatrix}$$

for an optimal objective value of 12.

(b) Now solve the new LP

$$A = \begin{bmatrix} 3 & 2 & 1 & 0 & 0 \\ 5 & 3 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ 16 \\ 5 \end{bmatrix}, \quad c = \begin{bmatrix} 5 \\ 1 \\ -12 \\ 0 \\ 0 \end{bmatrix}$$

$eps = 1e-6$ and $B = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \rightarrow$ THIS CORRESPONDS TO $x_0 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

with Kaihk's dual simplex code

$x_1 + x_2 - x_5 = 5$

$\therefore x_5 = 2 + 2 - 5$

$= -1$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}}_{A_B^{-1}} \begin{bmatrix} 10+\delta \\ 16 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2-3\delta \\ 2+5\delta \end{bmatrix}$$

Now x_1 and x_2 are basic as long as they are positive

i.e when δ lies in the following range $\boxed{-2/5 \le \delta \le 2/3}$

For δ within this range the basis B is unchanged, i.e, x_1 and x_2 are still the basic variables

The only thing that changes is the VALUE of these basic variables

However, if δ lies outside this range the basis B changes too.

however, also note that since A and c are unchanged, the dual optimal solution to LP (1) is still feasible in the new problem

(d) Changes in c

Suppose we add v to the 3rd component of c

We want to know the values of v which will change the optimal basis B to the previous LP (1)

Our new LP is

$$\text{Max} \quad 5x_1 + x_2 + (-12+v)x_3$$

$$\text{s.t} \quad \begin{aligned} 3x_1 + 2x_2 + x_3 &= 10 \\ 5x_1 + 3x_2 + \quad x_4 &= 16 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Note that only the coefficient of x_3 which is a NONBASIC variable in the optimal dictionary to the original LP (1) has changed

ie Only c_N has changed

Consider

$$c_N - A_N^T y = \begin{pmatrix} -12+v \\ 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -10 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} -12+v+10 \\ -7 \end{pmatrix} = \begin{pmatrix} -2+v \\ -7 \end{pmatrix}$$