ALTERNATE APPROACH USING THE PRIMAL/DUAL SIMPLEX CODE

(a) Solve the original LP

\[
A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ 16 \end{bmatrix}, \quad c = \begin{bmatrix} 5 \\ -12 \\ 0 \end{bmatrix}
\]

\[\varepsilon = 1e-6\] and \[B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}\]

with Kachh\'s revised simplex code

The optimal solution is

\[
X^* = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad Y^* = \begin{bmatrix} 7 \\ 10 \end{bmatrix}
\]

for an optimal objective value of 12.

(b) Now solve the new LP

\[
A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ 16 \\ 5 \end{bmatrix}, \quad c = \begin{bmatrix} 5 \\ -12 \\ 0 \end{bmatrix}
\]

\[\varepsilon = 1e-6\] and \[B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}\] \(\rightarrow\) This corresponds to \[X_0 = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix}\]

with Kachh\'s dual simplex code

\[
X_1 + X_2 - X_5 = 5
\]

\[
\therefore X_5 = 2 + 2 - 5
\]

\[
= -1
\]
\[
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} =
\begin{bmatrix}
-3 & 2 \\
5 & -3
\end{bmatrix}
\begin{bmatrix}
10 + 8 \\
16
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} =
\begin{bmatrix}
2 - 35 \\
2 + 58
\end{bmatrix}
\]

Now \(X_1\) and \(X_2\) are basic as long as they are positive

\[\text{I.e. when } S \text{ lies in the following range} \]

\[-2/5 \leq S \leq 2/3\]

For \(S\) within this range the basic \(B\) is unchanged, i.e., \(x_1\) and \(x_2\) are still the basic variables.

The only thing that changes is the value of these basic variables.

However, if \(S\) lies outside this range the basis \(B\) changes too.

However, also note that since \(A\) and \(c\) are unchanged, the dual optimal solution to LP (1) is still feasible in the new problem.
(d) Changes in $c$

Suppose we add $v$ to the 3rd component of $c$

We want to know the values of $v$ which will change the optimal basis $B$ to the previous LP.

Our new LP is

$$\text{Max } 5x_1 + x_2 + (-12 + v)x_3$$

$$\text{S.t. } 3x_1 + 2x_2 + x_3 = 10$$
$$5x_1 + 3x_2 + x_4 = 16$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Note that only the coefficient of $x_3$ which is a non-basic variable in the optimal dictionary to the original LP has changed.

Consider $C_N = A^T y = \begin{pmatrix} -12 + v \\ 0 \\ \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 10 \\ -10 \\ -7 \\ \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -12 + v + 10 \\ -7 \end{pmatrix} = \begin{pmatrix} 10 + v \\ -7 \end{pmatrix}$