If we add \( x_5 = -1 - 2x_3 + x_4 \) to the optimal dictionary of the original LP, we have

\[
\begin{align*}
  x_1 &= 2 + 3x_3 - 2x_4 \\
  x_2 &= 2 - 5x_3 + 3x_4 \\
  x_5 &= -1 - 2x_3 + x_4 \\
\end{align*}
\]

\[ 2 = 12 - 2x_3 - 7x_4 \]

This dictionary is PRIMAL INFEASIBLE but DUAL FEASIBLE

Construct the dual dictionary which is

\[
\begin{align*}
  s_3 &= 2 - 3s_1 + 5s_2 + 2s_5 \\
  s_4 &= 7 + 2s_1 - 3s_2 - s_5 \\
  -w &= -12 - 2s_1 - 2s_2 + s_5 \\
\end{align*}
\]

and apply the Simplex method to this dictionary

In the next iteration, \( s_5 \) ENTERS the basic while \( s_4 \) LEAVES

Continue until optimality
Changes in the rhs vector \( b \)

Suppose we add \( \delta \) to the first component of \( b \)

We want to know the values of \( \delta \) which will change the optimal basis \( B \) to the previous LP (2)

If the optimal basis changes, we want to know how to reoptimize the new LP using sensitivity analysis.

Our new LP is

\[
\begin{align*}
\text{Max} & \quad 5x_1 + x_2 - 12x_3 \\
\text{s.t.} & \quad 3x_1 + 2x_2 + x_3 = (10 + \delta) \\
& \quad 5x_1 + 3x_2 + x_4 = 16 \\
& \quad x_1, x_2, x_3, x_4 \geq 0
\end{align*}
\]

Note that changing \( b \) changes the value of the basic variables.

Since \( x_B = A_B^{-1} b \)

Since \( x_1 \) and \( x_2 \) were the basic variables in the previous optimal solution, we have
So, we can REOPTIMIZE using the dual Simplex method.

**EXERCISE:** Choose \( S = 1 \) (which is outside the range) and reoptimize with the dual Simplex method.

**ALTERNATE method using primal and dual Simplex models:**

(a) Solve the original LP

for \( x^* = \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \end{bmatrix} \) and \( y^* = \begin{bmatrix} -10 \\ 7 \\ 0 \end{bmatrix} \)

and an optimal objective value of 12.

(b) Now solve the new LP

\[
A = \begin{bmatrix} 3 & 2 & 10 \\ 5 & 3 & 0 \\ 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 11 \\ 16 \\ 1 \\ 0 \end{bmatrix}, \quad c = \begin{bmatrix} 5 \\ 1 \\ 12 \\ 0 \end{bmatrix}
\]

\( \varepsilon_p = 1 \times 10^{-6} \) and \( B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow x_0^* = \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix} \)

with Koehk's dual Simplex method.