If we add $x_5 = -1 - 2x_3 + x_4$ to the optimal dictionary of the original LP, we have

$$x_1 = 2 + 3x_3 - 2x_4$$
$$x_2 = 2 - 5x_3 + 3x_4$$
$$x_5 = -1 - 2x_3 + x_4$$
$$z = 12 - 2x_3 - 7x_4$$

This dictionary is PRIMAL INFEASIBLE but DUAL FEASIBLE

Construct the dual dictionary which is

$$s_3 = 2 - 3s_1 + 5s_2 + 2s_5$$
$$s_4 = 7 + 2s_1 - 3s_2 - s_5$$
$$-w = -12 - 2s_1 - 2s_2 + s_5$$

and apply the simplex method to this dictionary

In the next iteration, s_5 ENTERS the basis while s_4 LEAVES

Continue until optimality

(c) Changes in the rhs vector b

Suppose we add δ to the first component of b

We want to know the values of δ which will change the optimal basis B to the previous LP (1)

If the optimal basis changes, we want to know how to reoptimize the new LP using sensitivity analysis

Our new LP is

$$\text{Max} \quad 5x_1 + x_2 - 12x_3$$

$$\text{s.t.} \quad 3x_1 + 2x_2 + x_3 = (10 + \delta)$$

$$5x_1 + 3x_2 + x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Note that changing b changes the value of the basic variables

since $x_B = A_B^{-1} b$

Since x_1 and x_2 were the basic variables in the previous optimal solution we have

So, we can REOPTIMIZE using the dual simplex method

EXERCISE :- Choose $\delta = 1$ (which is outside the range) and reoptimize with the dual simplex method

ALTERNATE method using primal and dual simplex codes :-

(a) Solve the original LP

for $x^* = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ and $y^* = \begin{bmatrix} -10 \\ 7 \end{bmatrix}$

and an optimal objective value of 12.

(b) Now solve the new LP

$A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 11 \\ 16 \end{bmatrix}$, $C = \begin{bmatrix} 5 \\ 1 \\ -12 \\ 0 \end{bmatrix}$

$eps = 1e{-}6$ and $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow x_0^* = \begin{bmatrix} -1 \\ 7 \\ 0 \\ 0 \end{bmatrix}$

with Kartik's dual simplex method