The optimal dictionary is

$$x_1 = 2 + 3x_3 - 2x_4$$
$$x_2 = 2 - 5x_3 + 3x_4$$
$$z = 12 - 2x_3 - 7x_4$$

The optimal dual dictionary is

$$s_3 = 2 - 3s_1 + 5s_2$$
$$s_4 = 7 + 2s_1 - 3s_2$$
$$-w = -12 - 2s_1 - 2s_2$$

The optimal primal solution is

$$x^* = (2, 2, 0, 0)$$

The optimal dual solution is

$$y^* = (-10, 7)$$

and $s^* = (0, 0, 2, 7)$

(a) Suppose we add a new variable x_5

$c_5 = 1$ and $a_5 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$A_B^{-1} A_N = A_B^{-1} \left[a_3 \; a_4 \; a_5 \right]$$
$$= \left[A_B^{-1} a_3 \;\; A_B^{-1} a_4 \;\; A_B^{-1} a_5 \right]$$

We have $A_B^{-1} a_5$ (NEW NONBASIC COLUMN) $= \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Also $s_N = \left[s_3 \; s_4 \; s_5 \right]$
$$= \left[c_3 - A_3^T y ,\; c_4 - A_4^T y ,\; c_5 - A_5^T y \right]$$

We also have
$$(c_5 - A_5^T y) = \left(1 - (1 \; 1) \begin{pmatrix} -10 \\ 7 \end{pmatrix} \right)$$
$$= 4$$

$\therefore$ The new dictionary is

$$x_1 = 2 + 3x_3 - 2x_4 + x_5$$
$$x_2 = 2 - 5x_3 + 3x_4 - 2x_5$$
$$\overline{}$$
$$z = 12 - 2x_3 - 7x_4 + 4x_5$$

Since the coefficient of x_5 in the objective function row is POSITIVE, we are NOT OPTIMAL. We can however

(b) SUPPOSE we add a constraint

$x_1 + x_2 \geq 5$ to the original LP ①

We know that
$x_1 = 2 + 3x_3 - 2x_4$ and $x_2 = 2 - 5x_3 + 3x_4$
from the optimal dictionary to the original problem

$\therefore x_1 + x_2 = (2 + 3x_3 - 2x_4) + (2 - 5x_3 + 3x_4) \geq 5$

$\therefore (4 - 2x_3 + x_4) \geq 5$

OR $x_5 = -1 - 2x_3 + x_4$
(x_5 IS A slack variable in the new constraint)

When we introduce a new constraint in the primal, we introduce a new SURPLUS variable in the dual problem. Let s_5 be the new dual surplus variable

Our complementary variables are

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \Longleftrightarrow \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{pmatrix}$$