SENSITIVITY ANALYSIS

NOTES BY KARTIK

Consider

$$\text{Max } z = 5x_1 + x_2 - 12x_3$$

s.t.

$$\begin{aligned} 3x_1 + 2x_2 + x_3 \quad &= 10 \\ 5x_1 + 3x_2 \quad + \quad x_4 &= 16 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned} \longrightarrow (1)$$

THE DUAL problem is

$$\text{MIN } 10y_1 + 16y_2 \qquad \text{OR} \qquad \text{MIN } 10y_1 + 16y_2$$

$$\begin{aligned} \text{s.t } 3y_1 + 5y_2 &\geq 5 \\ 2y_1 + 3y_2 &\geq 1 \\ y_1 &\geq -12 \\ y_2 &\geq 0 \end{aligned} \qquad \begin{aligned} \text{s.t } 3y_1 + 5y_2 - s_1 &= 5 \\ 2y_1 + 3y_2 - s_2 &= 1 \\ y_1 - s_3 &= -12 \\ y_2 - s_4 &= 0 \end{aligned}$$

THE COMPLEMENTARY VARIABLES ARE

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \Longleftrightarrow \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix}$$

PRIMAL VARIABLES   DUAL SLACKS

The new LP is

$$\text{Max } 5x_1 + x_2 - 12x_3 + x_5 \longrightarrow 2$$

$$\text{s.t } \begin{aligned} 3x_1 + 2x_2 + x_3 + x_5 &= 10 \\ 5x_1 + 3x_2 \qquad + x_4 + x_5 &= 16 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

Let up update the optimal dictionary for the original LP 1

Note that the earlier solution $x^* = (2,2,0,0)$ together with $x_5 = 0$ is feasible in the new LP 2

Since the number of constraints has not changed, the size of the basis DOES NOT Change

On the other hand, we have a NEW NONBASIC VARIABLE x_5

We have $a_5 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Also $c_5 = 1$

The dictionary is

$$x_B = A_B^{-1} b - A_B^{-1} A_N x_N$$

$$z = c_B^T A_B^{-1} b + (c_N - A_N^T y)^T x_N$$

where $A_B = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$A_B^{-1} = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$

Also $B = \{1,2\}$
$N = \{3,4,5\}$

continue with the simplex method
In the next iteration x_5 enters and
x_2 LEAVES the basis

ALTERNATE APPROACH USING KARTIK'S
PRIMAL AND DUAL SIMPLEX MATLAB CODES

(a) Solve the original LP ①

$$A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ 16 \end{bmatrix}, \quad C = \begin{bmatrix} 5 \\ 1 \\ -12 \\ 0 \end{bmatrix}$$

$eps = 1e-6$ and $B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ x_3, x_4 ARE THE ORIGINAL BASIC VARIABLES

with Kartik's REVISED SIMPLEX code

The optimal solution is

$$x^* = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \text{ and } y^* = \begin{bmatrix} -10 \\ 7 \end{bmatrix}$$

for an objective value of 12

(b) Now solve the new LP ②

$$A = \begin{bmatrix} 3 & 2 & 1 & 0 & 1 \\ 5 & 3 & 0 & 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ 16 \end{bmatrix}, \quad C = \begin{bmatrix} 5 \\ 1 \\ -12 \\ 0 \\ 1 \end{bmatrix}$$

$eps = 1e-6$ and $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ x_1, x_2 ARE OUR BASIC VARIABLES

with Kartik's REVISED Simplex code again