MA/CSC 427-001: Introduction to Numerical Analysis I
Class Project
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1 Introduction

We will study the predator-prey problem in this project.

A mathematical model for the population dynamics of competing species, one a predator (lion) with population $y_L(t)$ and the other its prey (gazelle) $y_G(t)$ was developed in the early 1900s by A.J. Lotka and V. Volterra. Let b_G, b_L denote the birth rates and d_G, d_L denote the death rates for gazelles and lions, respectively. The rate of change of the lion and gazelle populations can be modeled by the following coupled system of first-order ordinary differential equations (ODEs):

$$\begin{aligned} \frac{dy_L}{dt} &= b_L y_L y_G - d_L y_L \\ \frac{dy_G}{dt} &= b_G y_G - d_G y_G y_L. \end{aligned} \tag{1}$$

The initial conditions are $y_G(0) = 3000$ and $y_L(0) = 500$. Moreover, the coefficients in the model are $b_G = 1.1/\text{yr}$, $b_L = 0.00025/\text{yr}$, $d_G = 0.0005/\text{yr}$, and $d_L = 0.7/\text{yr}$. Determine the populations $y_L(t)$ of lions and $y_G(t)$ of gazelles from $t = 0$ to $t = 25$ years.

A short description of the model (1) is now in order: We assume that there is plenty of food available for the gazelles, so the birthrate of the gazelles follows the Malthusian or exponential law, i.e., the birthrate of gazelles is $b_G y_G$, where b_G is a positive constant. The death rate of the gazelles depends on the number of interactions between the gazelles and the lions. This is modeled by the expression $d_G y_G y_L$, where d_G is a positive constant. Therefore, the rate of change in the population of the gazelles per unit time is given by $\frac{dy_G}{dt} = b_G y_G - d_G y_G y_L$. Assuming that the lions depend entirely on the gazelles for their food, the birthrate of the lions depends on the number of their interactions with the gazelles, i.e., the birthrate of the lions is given by $b_L y_L y_G$, where b_L is a positive constant. The death rate of the lions is given by $d_L y_L$ because without food the lions would die at a rate proportional to the current population of lions. Hence, the rate of change in the population of the lions per unit time is given by $\frac{dy_L}{dt} = b_L y_L y_G - d_L y_L$. Combining the two ODEs, we get the Lotka-Volterra model (1).

Note that (1) represents a system of nonlinear ODEs that are not explicitly solvable. The purpose of this project is to numerically solve this nonlinear system on a computer.

2 Details of the project

1. You will work in a group consisting of 3 students. The entire project is due in class on Wednesday, the 5th of December, 2007. If you have any question on any parts of the project, please send me email or come and see me.

2. Read section 5.9 in Burden and Faires carefully. This section develops a Runge-Kutta method for solving a system of ordinary differential equations.

3. You will write a computer program in MATLAB that implements Algorithm 5.7 on page 316 to solve an initial value problem involving m first order ODEs (5.44) and m initial value conditions (5.45) over the time interval $a \leq t \leq b$. The input variables to the algorithm are: a) the endpoints a and b, b) the number of ODEs m, c) the number of values $(N+1)$ of t, d) a function myfunc that evalutes the vector function $f(t, y) = [f_1(t, y), \ldots, f_m(t, y)]$ at (t, y), and e) the initial conditions $\alpha_1, \ldots, \alpha_m$. The algorithm will return a) the vector t containing the $(N+1)$ time values, and b) the matrix W with $(N+1)$ rows and m columns, whose ith column contains the approximate solution w_i at the $(N+1)$ values of t.

4. Solve the initial value problem (1) with your computer program to find the populations of lions and gazelles from $t = 0$ to $t = 25$. Plot the two populations versus time and also versus each other. What do you observe?

5. Solve the initial value problem (1) using MATLABs ODE solver *ode45*. Type *help ode45* in MATLAB for more details. Plot the two populations versus time and also versus each other. Compare your results with those in part (4).

6. Each group will submit a 3-4 page report that describes the predator-prey model, the details of the Runge-Kutta algorithm to solve an initial value problem involving a system of first order ODEs, your MATLAB program for part (3), and the numerical results and plots with your algorithm and MATLABs ODE solver *ode45* on the model (1). You should also briefly summarize your findings in the report. Finally, each member of the group should clearly indicate his/her contribution on the project.