MA/CSC 427-001: Introduction to Numerical Analysis I
Review questions for midterm exam
Instructor: *Dr. Kartik Sivaramakrishnan*

INSTRUCTIONS

The questions on the midterm exam will similar to these review problems. Work out each problem in sufficient detail with your classmates. If you have any questions, please send me an email. I will post the solutions to selected problems on the course webpage next Friday.

1. Floating point representation: What is the gap between 2 and the first IEEE single format number larger than 2? What is the gap between 1024 and the first IEEE single format number larger than 1024?

2. Finite difference formulas for numerical differentiation:

   (a) Show that the truncation error in the following 3 point forward difference formula

   $$f'(x_i) = \frac{-3f(x_i) + 4f(x_{i+1}) - f(x_{i+2})}{2h}$$

   is $O(h^2)$.

   (b) Show that the truncation error in the following 3 point forward difference formula

   $$f''(x_i) = \frac{f(x_i) - 2f(x_{i+1}) + f(x_{i+2})}{h^2}$$

   is $O(h)$.

3. Elements of numerical integration: Derive the following formula for Simpson's rule

   $$\int_a^b f(x)\mathrm{dx} = \frac{h}{3}(f(x_0) + 4f(x_1) + f(x_2)) - \frac{h^5}{12}(\frac{1}{3}f^4(\xi_2) - \frac{1}{5}f^4(\xi_1))$$

   where $x_0 = a$, $x_2 = b$, and $h = \frac{(b-a)}{2}$. Show all your steps. What is the degree of accuracy of Simpson's rule? Why?

4. Composite numerical integration: Exercise Set 4.4, Page 204, Problem 13(b).

5. Order of convergence: Exercise Set 2.4, Page 82, Problems 6, 8, and 11.

6. Newton's divided differences: Exercise Set 3.2, Page 129, Problem 19.

7. Polynomial interpolation: Find the Lagrange and Newton interpolating polynomials for the following data

x	$f(x)$
-2	0
0	1
1	-1

Write both polynomials in the form $a+bx+cx^2$ in order to verify that they are identical.

8. Halley's method for solving the nonlinear equation $f(x) = 0$ uses the following iterate formula

$$x_{n+1} = x_n - \frac{f(x_n)f'(x_n)}{(f'(x_n))^2 - \frac{f(x_n)f''(x_n)}{2}}$$

in the n iteration. Show that this formula results when Newton's method is applied to the function
$g(x) = \frac{f}{\sqrt{f'(x)}}$.