

MA 427-001: Introduction to Numerical Analysis I  
Final Exam Review  
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## INSTRUCTIONS

The questions on the final will be similar to these review problems. Work out each problem in sufficient detail. If you have any questions, please send me an email. I will post the solutions to selected problems on the course webpage on Friday.

1. Exercise Set 11.1, Page 648, Problem 1. Do part 1(a) only!
2. Consider the following second order ODE:

$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 2x, \quad 0 \leq x \leq 1, \quad y(0) = 1 \quad \text{and} \quad y(1) = 1. \quad (1)$$

- (a) Using the central difference formulas

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1}))}{2h}$$
$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1}))}{h^2}$$

for approximating the 1st and 2nd order derivatives in (1) respectively, discretize the ODE and write it in a form suitable for solution with the finite difference method.

- (b) Write down the tridiagonal system of linear equations for the finite difference method in matrix form for the following two cases: (1)  $h = 0.5$  and (2)  $h = \frac{1}{3}$ .
- (c) Write down your MATLAB commands and functions to solve (1) using MATLAB's boundary value solver *bvp4c*?
3. Consider the following boundary value problem

$$\frac{d^2y}{dx^2} + ay + by^4 = 0, \quad 0 \leq x \leq 1, \quad \frac{dy}{dx}(x=0) = 0, \quad y(1) = 1 \quad (2)$$

where  $a$  and  $b$  are constants.

- (a) Discretize the second order ODE using: (a) the central difference formula for approximating the 2nd derivative in (2) and (b) the 2nd order forward difference formula

$$f'(x_i) = \frac{-3f(x_i) + 4f(x_{i+1}) - f(x_{i+2}))}{2h}$$

for approximating the 1st derivative in the boundary condition  $\frac{dy}{dx}(0) = 0$ . What is the system of nonlinear equations when  $h = 0.5$ ?

- (b) We will solve the nonlinear system of equations when  $h = 0.5$  using Newton's method from Section 10.2 of Burden and Faires. Set up the linear system of equations that are solved in each iteration of Newton's method. You do not have solve these equations.
- (c) Write down your MATLAB commands and functions to solve (2) using MATLAB's boundary value solver *bvp4c*?
4. Consider the following first order ODE:

$$\frac{dy}{dx} = yx - x^2, \quad 0 \leq x \leq 1.8, \quad y(0) = 1. \quad (3)$$

- (a) Solve by hand with Euler's explicit method using  $h = 0.6$ .
- (b) Solve by hand with modified Euler's method using  $h = 0.6$ .
- (c) Solve by hand using the 4th order Runge-Kutta method using  $h = 0.6$ .
5. Write the following system of two 2nd order ODEs as a system of four 1st order ODEs:

$$\frac{d^2x}{dt^2} = -\frac{\gamma}{m} \frac{dx}{dt} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}, \quad \frac{d^2y}{dt^2} = -g - \frac{\gamma}{m} \frac{dy}{dt} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

where  $g$ ,  $m$ , and  $\gamma$  are given constants.

6. Consider the following system of two ODEs:

$$\frac{dx}{dt} = xt - y, \quad \frac{dy}{dt} = yt + x, \quad 0 \leq t \leq 1.2, \quad x(0) = 1, \quad \text{and} \quad y(0) = 0.5. \quad (4)$$

- (a) Solve by hand with Euler's explicit method using  $h = 0.4$ .
- (b) Solve by hand using the 4th order Runge-Kutta method using  $h = 0.4$ .
7. Write down your MATLAB commands and functions to solve the following initial value problems in Burden and Faires using MATLAB's ODE solver *ode45*:
- (a) Exercise Set 5.9, Page 323, Problem 2(c).
- (b) Exercise Set 5.9, Page 323, Problem 3(d).

8. A particular finite difference formula for the first derivative of a function is

$$f'(x_i) = \frac{-f(x_{i+3}) + 9f(x_{i+1}) - 8f(x_i)}{6h}$$

where the points  $x_i, x_{i+1}, x_{i+2}$ , and  $x_{i+3}$  are all equally spaced with step size  $h$ . What is the order of the truncation error? Give reasons for your answer.

9. The centroid of a circular sector is given by

$$\bar{x} = \frac{2r \sin \theta}{3\theta}.$$

Find  $\theta$  such that  $\bar{x} = \frac{r}{2}$  using

- (a) Use bisection method with  $a = 1$  and  $b = 2$ .
- (b) Use secant method with  $x_1 = 1$  and  $x_2 = 1$ .
- (c) Use Newton's method with  $x_1 = 1$ .

Carry out the first two iterations in each case.

10. Consider the system of nonlinear equations

$$\begin{aligned} 4x^2 - y^3 &= -28 \\ 3x^3 + 4y^2 &= 145. \end{aligned}$$

Use Newton's method to solve the nonlinear system with  $x = 1$  and  $y = 1$ . Carry out the first two iterations.