HOMEWORK # 5
SOLUTIONS ①

(PREPARED BY KARTIK)

① EXERCISE 7.5
Page 384, 37

Solve $ty'' - 2y' + ty = 0$
with $y(0) = 1$ and $y'(0) = 0$

$$\therefore L\{ty''\} - 2L\{y'\} + L\{ty\} = 0$$

$$-\frac{d}{ds}\left(s^2Y(s) - sy(0) - y'(0)\right) - 2\left(sY(s) - y(0)\right) - (1)\frac{d}{ds}Y(s) = 0$$

where $L\{y(t)\} = Y(s)$

Substituting the initial value conditions gives

$$-\frac{d}{ds}\left(s^2Y(s) - s\right) - 2\left(sY(s) - 1\right) - \frac{d}{ds}Y(s) = 0$$

$$\therefore -\left(2sY(s) + s^2Y'(s) - 1\right) - 2\left(sY(s) - 1\right) - Y'(s) = 0$$

$\therefore \; -\left(2sY(s) + s^2Y'(s) - 1\right) - 2\left(sY(s) - 1\right.$
$- Y'(s) = 0$

$\therefore \; -(s^2+1)\,Y'(s) - 4sY(s) + 3 = 0$

$\therefore \; (s^2+1)\,Y'(s) + 4sY(s) = 3$

$$\therefore \; \boxed{Y'(s) + \frac{4s}{(s^2+1)}\,Y(s) = \frac{3}{(s^2+1)}}$$

This is a linear first order different
equation with Y (dep var) and s
(ind var).

Use discussion in Sec 2-3 to solve equation

(1) $\mu(s) = e^{\int \frac{4s}{(s^2+1)}ds}$

$= e^{2\log|s^2+1|}$

$= e^{\log(s^2+1)^2}$

$= (s^2+1)^2$

$\int \frac{4s}{(s^2+1)}\,ds$
Let $s^2+1 =$
$2s\,ds = d$
$\int \frac{2dt}{t} = 2\log$
$= 2\log|s^2$

(2) $Y(s) = \frac{1}{\mu(s)}\left[\int \mu(s)\frac{3}{(s^2+1)}ds + c\right]$

$\therefore \; Y(s) = \frac{1}{(s^2+1)^2}\left[\int (s^2+1)^2\frac{3}{(s^2+1)}ds + c\right.$

3

$$\therefore Y(s) = \frac{1}{(s^2+1)^2}\left[\int 3(s^2+1)\,ds + c\right]$$

$$= \frac{1}{(s^2+1)^2}\left[\frac{3s^3}{3} + 3s + c\right]$$

$$\boxed{Y(s) = \frac{s^3+3s}{(s^2+1)^2} + \frac{c}{(s^2+1)^2}} \rightarrow (11)$$

$$\therefore y(t) = L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{(s^3+3s)}{(s^2+1)^2} + \frac{c}{(s^2+1)^2}\right\}$$

$$= L^{-1}\left\{\frac{(s^3+3s)}{(s^2+1)^2}\right\} + c\,L^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}$$

(Since c is a constant)

Now

$$L^{-1}\left\{\frac{(s^3+3s)}{(s^2+1)^2}\right\} = L^{-1}\left\{\frac{As+B}{(s^2+1)} + \frac{Cs+D}{(s^2+1)^2}\right\}$$

$$= L^{-1}\left\{\frac{(As+B)(s^2+1) + (Cs+D)}{(s^2+1)^2}\right\}$$

$$= L^{-1}\left\{\frac{(As+B)(s^2+1) + (Cs+D)}{(s^2+1)^2}\right\}$$

$$= L^{-1}\left\{\frac{As^3 + (A+C)s + Bs^2 + (B+D)}{(s^2+1)^2}\right\}$$

$$\therefore L^{-1}\left\{\frac{s^3+3s}{(s^2+1)^2}\right\} = L^{-1}\left\{\frac{As^3 + Bs^2 + (A+C)s + (B+D)}{(s^2+1)^2}\right\}$$

$\therefore A = 1 \qquad A + C = 3 \qquad B = D = 0$

$\therefore C = 2$

$$\therefore L^{-1}\left\{\frac{s^3+3s}{(s^2+1)^2}\right\} = L^{-1}\left\{\frac{s}{(s^2+1)} + \frac{2s}{(s^2+1)^2}\right\}$$

$$= \cos t + 2L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$$

Now $L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = L^{-1}\left\{\frac{s}{(s^2+1)} \cdot \frac{1}{(s^2+1)}\right\}$

$= \cos t * \sin t$ (CONVOLUTION) $= \int^{t} \cos(t-u)\sin(u)\,du$

5

$$\therefore \mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \int_0^t \cos(t-u)\sin(u)\,du$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(B+A) + \sin(B-A)\right]$$

$$\int_0^t \cos(t-u)\sin(u)\,du$$

$$= \int_0^t \frac{1}{2}\left[\sin(t) + \sin(2u-t)\right]du$$

$$= \frac{1}{2}\left[\sin(t)u - \frac{\cos(2u-t)}{2}\right]_{u=0}^{u=t}$$

$$= \frac{1}{2}\left[t\sin t - \cancel{\frac{1}{2}\cos(t) + \frac{\cos(-t)}{2}}\right]$$

Since $\cos(-t) = \cos(t)$

$$= \frac{1}{2}t\sin t$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{s^3+3s}{(s^2+1)^2}\right\} = \cos t + \frac{2}{2}t\sin t$$

We will use the hint to compute/write

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}(t) = \frac{1}{2}(\sin t - t\cos t)$$

$$\therefore \boxed{\begin{aligned} y(t) &= \cos t + \frac{2}{2} t\sin t + \frac{c}{2}(\sin t - t\cos t \\ &= \cos t + t\sin t + \frac{c}{2}(\sin t - t\cos t \end{aligned}}$$

2 EXERCISE 7-6

Problem 19

Consider

$$I''(t) + 2I'(t) + 2I(t) = g(t)$$

$$I(0) = 10, \quad I'(0) = 0$$

and

$$g(t) = \begin{cases} 20, & 0 < t < 3\pi \\ 0, & 3\pi < t < 4\pi \\ 20, & 4\pi < t \end{cases}$$

Determine $I(t)$.

Step 1:- write $g(t)$ in terms of step functions

$$g(t) = 20 + (0-20)\, u(t-3\pi) + (20-0)\, u(t-4\pi)$$

$$= 20 - 20u(t-3\pi) + 20u(t-4\pi)$$

$$\therefore G(s) = L\{g(t)\}$$

$$= \frac{20}{s} - \frac{20e^{-3\pi s}}{s} + \frac{20e^{-4\pi s}}{s}$$

Step 2:- Take Laplace transform on both sides of the diff equation and substitute the value of $G(s)$ from step 1.

we have

$$(s^2 I(s) - sI(0) - I'(0)) + 2(sI(s) - I(0)) + 2I(s)$$

$$= \frac{20}{s} - \frac{20e^{-3\pi s}}{s} + \frac{20e^{-4\pi s}}{s}$$

where $I(s) = L\{I(t)\}$

8

$$\therefore \quad (s^2+2s+2)\, I(s) - 10s - 20 = \frac{20}{s} - \frac{20e^{-3\pi s}}{s} + \frac{20e^{-4\pi s}}{s}$$

$$\therefore \quad I(s) = \frac{10s+20}{(s^2+2s+2)} + \frac{20}{s/((s+1)^2+1)} - \frac{20e^{-3\pi s}}{s/((s+1)^2+1)} + \frac{20e^{-4\pi s}}{s/((s+1)^2+1)}$$

Now $I(t) = L^{-1}\{I(s)\}$

$$= L^{-1}\left\{\frac{10s+20}{s^2+2s+2}\right\} + L^{-1}\left\{\frac{20}{/((s+1)^2+1)}\right\} - L^{-1}\left\{\frac{20e^{-3\pi s}}{s/((s+1)^2+1)}\right\} + L^{-1}\left\{\frac{20e^{-4\pi s}}{s/((s+1)^2+1)}\right\}$$

Now

$$L^{-1}\left\{\frac{10s+20}{((s+1)^2+1)}\right\} = L^{-1}\left\{\frac{10(s+1)}{((s+1)^2+1)}\right\} + L^{-1}\left\{\frac{10}{((s+1)^2+1)}\right\} = 10e^{-t}(\cos t + \sin t$$

Consider

$$L^{-1}\left\{\frac{20}{s((s+1)^2+1)}\right\} = L^{-1}\left\{\frac{A}{s}+\frac{(Bs+C}{((s+1)^2+}\right.$$

$$= L^{-1}\left\{\frac{A(s^2+2s+2)+Bs^2+Cs}{s((s+1)^2+1)}\right.$$

$$= L^{-1}\left\{\frac{(A+B)s^2+(2A+C)s+2A}{s((s+1)^2+1)}\right\}$$

$\therefore 2A = 20$ $\qquad A+B=0 \qquad 2A+C=0$

$A = 10$ $\qquad B=-A \qquad C=-2A$

$B=-10 \qquad =-20$

$$\therefore L^{-1}\left\{\frac{20}{s((s+1)^2+1)}\right\} = L^{-1}\left\{\frac{10}{s}\ \frac{-10s}{((s+1)^2+1)}\ \frac{-20}{((s+1)^2}\right.$$

$$= L^{-1}\left\{\frac{10}{s}\right\} - L^{-1}\left\{\frac{10s+20}{((s+1)^2+1)}\right\}$$

$$= 10 - 10e^{-t}(\cos t + \sin t)$$

Now

$$L^{-1}\left\{\frac{20e^{-3\pi s}}{s((s+1)^2+1)}\right\} = (10-10e^{-(t-3\pi)}(\cos(t-3\pi) + \sin(t-3\pi))$$

10

and

$$\mathcal{L}^{-1}\left\{\frac{20e^{-4\pi s}}{s((s+1)^2+1)}\right\}$$

$$= \left(10 - 10e^{-(t-4\pi)}\left(\cos(t-4\pi) + \sin(t-4\pi)\right)\right) u(t-4\pi)$$

where we have used the fact that

$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\,u(t-a)$$

where $\mathcal{L}^{-1}\{F(s)\} = f(t)$

(Theorem 8 on page 387)

$\therefore$ $I(t)$

$$= 10e^{-t}(\cos t + \sin t)$$

$$+ \left(10 - 10e^{-t}(\cos t + \sin t)\right)$$

$$- \left(10 - 10e^{-(t-3\pi)}\left(\cos(t-3\pi) + \sin(t-3\pi)\right)\right) u(t-3\pi$$

$$+ \left(10 - 10e^{-(t-4\pi)}\left(\cos(t-4\pi) + \sin(t-4\pi)\right)\right) u(t-4$$

$\therefore$

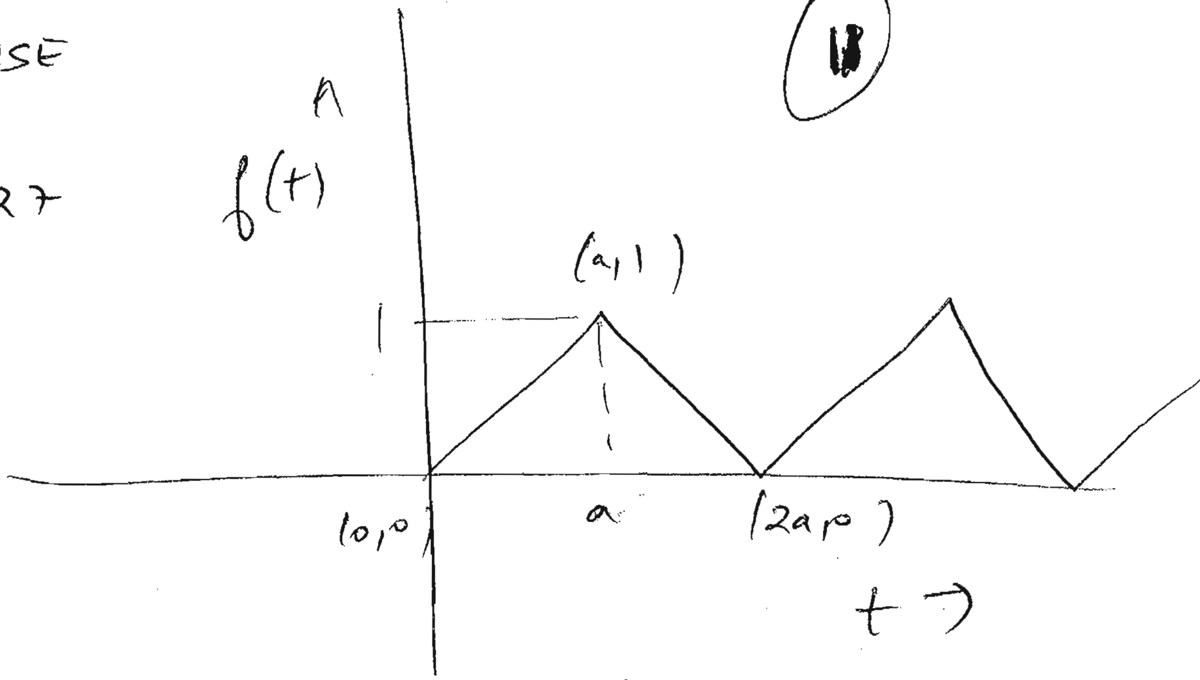
$$\boxed{I(t) = 10 - 10\,u(t-3\pi)\left(1 + e^{-(t-3\pi)}(\cos t + \sin t)\right) + 10\,u(t-4\pi)\left(1 - e^{-(t-4\pi)}(\cos t + \sin t)\right)}$$

Since $\cos(t-3\pi) = -\cos t$, $\sin(t-3\pi) = -\sin t$

$\cos(t-4\pi) = \cos t$ and $\sin(t-4\pi) = \sin t$

3

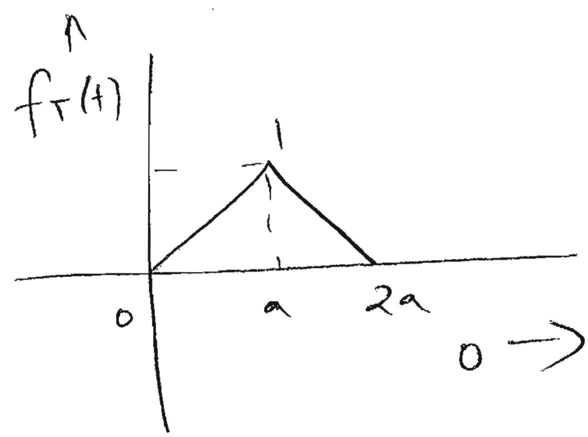
EXERCISE
7.6
PROBLEM 27



Find the laplace transform of $f(t)$.

The period of $f(t)$ is $\boxed{T = 2a}$

Consider
$f_T(t)$



$$f_T(t) = \frac{t}{a}, \quad 0 < t < a$$ → st line joining (0,0) and (a,1)

$$= -\frac{1}{a}(t-2a), \quad a < t < 2a$$ ↳ st line joining (a,1) and (2a,0)

$$= 0 \quad \text{otherwise}$$

$$\therefore f_T(t) = \frac{t}{a}, \quad 0<t<a$$

$$= \frac{-1}{a}(t-2a), \quad a<t<2a$$

$$= 0 \quad \text{otherwise}$$

$$\therefore f_T(t) = \frac{t}{a} - \frac{2}{a}(t-a)\,u(t-a) + \frac{1}{a}(t-2a)\,u(t-2a)$$

(IN TERMS OF UNIT STEP functions).

Note that $f_T(t)$ is actually a continuous function and is also defined at $t=a, 2a$.

However, one can also write a CONTINOUS function in terms of step functions.

$$\therefore F_T(s) = L\{f_T(t)\}$$

$$= L\left\{\frac{t}{a} - \frac{2}{a}(t-a)\,u(t-a) + \frac{1}{a}(t-2a)\,u(t-2a)\right\}$$

$\therefore F_T(s)$

$$= L\left\{\frac{t}{a}\right\} - \frac{2}{a} L\left\{(t-a)\,u(t-a)\right\}$$

$$+ \frac{1}{a} L\left\{(t-2a)\,u(t-2a)\right\}$$

Now $\quad L\left\{f(t-a)\,u(t-a)\right\} = e^{-as} F(s)$

where $\quad F(s) = L\left\{f(t)\right\}$

(see Theorem 8 on page 387)

$$\therefore F_T(s) = \frac{1}{as^2} - \frac{2}{a} e^{-as} L\{t\}$$

$$+ \frac{1}{a} e^{-2as} L\{t\}$$

$$= \frac{1}{as^2} - \frac{2}{a}\frac{e^{-as}}{s^2} + \frac{1}{a}\frac{e^{-2as}}{s^2}$$

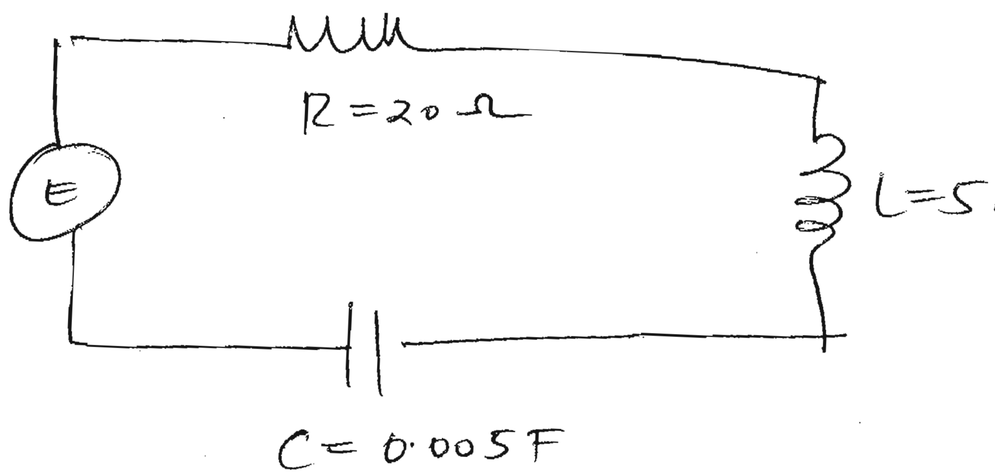
$$= \frac{1}{as^2}\left(1 - 2e^{-as} + e^{-2as}\right)$$

$$= \frac{1}{as^2}\left(1 - e^{-as}\right)^2$$

$$\therefore F(s) = \frac{F_T(s)}{1-e^{-sT}} = \frac{F_T(s)}{1-e^{-2as}} = \frac{1}{as^2}\frac{\left(1-e^{-as}\right)^2}{\left(1+e^{-as}\right)\left(1-e^{-as}\right)}$$

$$\therefore F(s) = \frac{1}{as^2} \; \frac{(1-e^{-as})}{(1+e^{-as})}$$

(4)
EXERCISE
7.7, Problem 29



$a = -1$ and $b = 8$

Consider

$$L I''(t) + R I'(t) + \frac{1}{C} I(t) = e(t)$$

$$I(0) = a \quad , \quad I'(0) = b$$

$$\therefore 5 I''(t) + 20 I'(t) + 200 I(t) = e(t)$$

$$I(0) = -1 \quad , \quad I'(0) = 8$$

We will find $I(t)$ in two steps.

15

(1) Step 1:– Find the impulse response $h(t)$ for the system

where $H(s) = L\{h(t)\}$

$= \dfrac{I(s)}{E(s)}$ where the initial conditions are assumed to be zero.

In our case we have

$5I''(t) + 20I'(t) + 200I(t) = e(t)$

$\therefore (5s^2 + 20s + 200)I(s) = E(s)$

$\therefore H(s) = \dfrac{1}{(5s^2+20s+200)}$

$= \dfrac{1}{5(s^2+4s+40)} = \dfrac{1}{5((s+2)^2+6^2)}$

$= \dfrac{1}{30}\dfrac{6}{((s+2)^2+$

$\therefore \boxed{h(t) = \dfrac{1}{30}e^{-2t}\sin 6t}$

Step 2 :- Find the solution $I_k(t)$ to

$$5I''(t) + 20I'(t) + 200I(t) = 0$$ (18)

with $I(0) = -1$ and $I'(0) = 8$.

Char equation

ie $\lambda^2 + 4\lambda + 40 = 0$

$$\lambda = \frac{-4 \pm \sqrt{16 - 4(40)}}{2}$$

$$= \frac{-4 \pm \sqrt{-144}}{2}$$

$$= -2 \pm 6i \qquad \begin{matrix} \alpha = -2 \\ \beta = 6 \end{matrix}$$

$$\therefore I(t) = c_1 e^{-2t}\cos(6t) + c_2 e^{-2t}\sin(6t)$$

$$I'(t) = -2c_1 e^{-2t}\cos(6t) - 6c_1 e^{-2t}\sin(6t)$$
$$- 2c_2 e^{-2t}\sin(6t) + 6c_2 e^{-2t}\cos(6t)$$

$I(0) = -1$ gives $\boxed{c_1 = -1}$

$I'(0) = -2c_1 + 6c_2 = 8$

$\therefore 2 + 6c_2 = 8 \quad \boxed{c_2 = 1}$

$\therefore I_R(t)$

$$= -e^{-2t}\cos(6t) + e^{-2t}\sin(6t)$$

By Theorem 12 on page 404 of the book we have

$$I(t) = (h * e)(t)$$
$$- e^{-2t}\cos(6t) + e^{-2t}\sin(6t)$$

$$\therefore \boxed{I(t) = \frac{1}{30}\int_0^t e(t-v)\cdot e^{-2v}\sin(6v)\,dv - e^{-2t}\cos(6t) + e^{-2t}\sin(6t)}$$

(5)

EXERCISE 9.1
PROBLEM 13

$$x'' - 3x' + t^2 y - (\cos t)x = 0$$
$$y''' + y'' - tx' + y' + e^t x = 0$$

let
$$x_1 = x$$
$$x_2 = x' = x_1'$$
$$x_3 = y$$
$$x_4 = y' = x_3'$$
$$x_5 = y'' = x_4'$$

The two diff equation are

$$x_2' - 3x_2 + t^2 x_3 - (\cos t)x_1 =$$
$$x_5' + x_5 - tx_2 + x_4 + e^t x_1 = 0$$

∴ We have

$$x_1' = x_2$$
$$x_2' = (\cos t)\, x_1 + 3x_2 - t^2 x_3$$
$$x_3' = x_4$$
$$x_4' = x_5$$
$$x_5' = -e^t x_1 + t x_2 - x_4 - x_5$$

Let $x' = \begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_5' \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_5 \end{bmatrix}$

$$\therefore\ x' = \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ (\cos t) & 3 & -t^2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -e^t & t & 0 & -1 & -1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$\therefore\ x' = Ax$$

(6)

(19)

EXERCISE 9.2

PROBLEM 13

Consider

$$2x_1 - 3x_2 = \ell x_1$$
$$x_1 - 2x_2 = \ell x_2$$

when $\ell = 2$ and 1 respectively

Consider the case when $\ell = 2$ first

$$\therefore \cancel{2x_1} - 3x_2 = \cancel{2x_1}$$
$$x_1 - 2x_2 = 2x_2$$

$\therefore -3x_2 = 0$

$x_2 = 0$

The 2nd equation gives $x_1 = 4x_2$

$\therefore x_1 = 0$

$\therefore$ The unique solution is $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Now consider the case when $\ell = 1$

$$\therefore 2x_1 - 3x_2 = x_1 \implies x_1 - 3x_2 = 0$$
$$x_1 - 2x_2 = x_2 \qquad\quad x_1 - 3x_2 = 0$$

In reality there is only one equation $x_1 - 3x_2 = 0$

We have one degree of freedom here, i.e we can choose any value for x_2 (for instance)

20

however $x_1 = 3x_2$

$\therefore$ Any solution

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

For diff values of x_2 we obtain diff solutions

$\therefore$ In this case we have an infinite no of solutions

7

EXERCISE 9.3
PROBLEM 30

Consider

$$A = \begin{bmatrix} 4 & -2 & 2 \\ -2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

(a) Compute the inverse of A

$[A \mid I]$

$$= \left[\begin{array}{ccc|ccc} 4 & -2 & 2 & 1 & 0 & 0 \\ -2 & 4 & 2 & 0 & 1 & 0 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_1 = 1/4\, R_1$ gives

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -1/2 & 1/2 & 1/4 & 0 & 0 \\ -2 & 4 & 2 & 0 & 1 & 0 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = R_2 + 2R_1$ and $R_3 = R_3 - 2R_1$ gives

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -1/2 & 1/2 & 1/4 & 0 & 0 \\ 0 & 3 & 3 & 1/2 & 1 & 0 \\ 0 & 3 & 3 & -1/2 & 0 & 1 \end{array}\right]$$

$R_2 = 1/3\, R_2$ gives

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 1 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 3 & 3 & -\frac{1}{2} & 0 & 1 \end{array}\right)$$

$R_1 = R_1 + \frac{1}{2} R_2 \qquad R_3 = R_3 - 3R_2$ gives

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{1}{3} & \frac{1}{6} & 0 \\ 0 & 1 & 1 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{array}\right)$$

$\frac{1}{4} \times 3 +$

$\frac{4}{12} =$

We cannot proceed any further.
Our eventual aim was to get the identity matrix I on the L.H.S
However we cannot get a 1 in pos (3,3) using elementary row operations

$\therefore$ The row reduction procedure fails to produce the inverse of A.

(b) det A

$$= 4(16-4) + 2(-8-4) + 2(-4-8)$$

$$= 48 - 24 - 24 = 0$$

(c) Find a nontrivial soln x to $Ax = 0$.

$Ax = 0$ can be put in the following form:

$$\left[\begin{array}{ccc|c} 4 & -2 & 2 & 0 \\ -2 & 4 & 2 & 0 \\ 2 & 2 & 4 & 0 \end{array}\right]$$

Using elementary row operations (as in part (a)) this reduces to

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Since the rhs is zero it is unaffected under the row operations performed.

$\therefore$ We have

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Note :- Performing the row operations both on the lhs, i.e on the coefficient matrix, as well as the rhs leaves the solution X unchanged.

$\therefore$ We have

$$x_1 + x_3 = 0 \qquad \Rightarrow \qquad x_1 = -x_3$$
$$x_2 + x_3 = 0 \qquad\qquad x_2 = -x_3$$
$$0 = 0$$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

Choosing $x_3 = 1$ gives

$$X = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

This is a non-trivial soln to $AX = 0$.

(d) Note

$$Ax = x_1 a_1 + x_2 a_2 + x_3 a_3$$

where a_1, a_2, a_3 are the 3 column vectors of A.

$\therefore$ The desired scalars c_1, c_2, c_3 are the coefficients x_1, x_2, x_3 computed in part (c)

i.e $c_1 = -1$, $c_2 = -1$ and $c_3 = 1$