CAAM 664 SDP Weighting Problem

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Here is the solution to the "Weighting Problem". Assume $X, S, W \in \mathcal{S}^n_{++}$. To solve this, I only had to use Spectral Factorization.

Spectral Factorization : for $A \in \mathcal{S}^n_{++}$, $\exists$ positive definite $A^{1/2}$ such that $A = A^{1/2}A^{1/2}$. Note that the matrix $A^{1/2}$ is merely $Pdiag(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_n})P^T$ in the Spectral factorization $A = P\Lambda P^T$, where $\Lambda = diag(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_n})$.

$$
\begin{aligned}
W^{-1}XW^{-1} &= S && (1)\\
W^{-1}X^{1/2}X^{1/2}W^{-1} &= S && (2)\\
X^{1/2}W^{-1}X^{1/2}X^{1/2}W^{-1}X^{1/2} &= X^{1/2}SX^{1/2} && (3)\\
(X^{1/2}W^{-1}X^{1/2})^2 &= X^{1/2}SX^{1/2} && (4)\\
(X^{1/2}W^{-1}X^{1/2})^2 &= (X^{1/2}SX^{1/2})^{1/2}(X^{1/2}SX^{1/2})^{1/2} && (5)\\
(X^{1/2}W^{-1}X^{1/2})^2 &= ((X^{1/2}SX^{1/2})^{1/2})^2 && (6)\\
X^{1/2}W^{-1}X^{1/2} &= (X^{1/2}SX^{1/2})^{1/2} && (7)\\
W^{-1} &= X^{-1/2}(X^{1/2}SX^{1/2})^{1/2}X^{-1/2} && (8)\\
W &= X^{1/2}(X^{1/2}SX^{1/2})^{-1/2}X^{1/2} && (9)
\end{aligned}
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