# CAAM 353 : Computational Numerical Analysis
## Homework 6
Instructor: *Dr. Kartik Krishnan*

## INSTRUCTIONS

Due in class on April 24th, 2003. No late homeworks will be accepted.

1. Given Runge's function

$$f(t) \quad = \quad \frac{1}{1+25t^2}$$

on the interval $[-1, 1]$.

(a) Compute 10th and 20th degree polynomial interpolants to Runge's function, using $n = 11$, and 21 equally spaced points on the interval $[-1, 1]$.

(b) Repeat part (a) using the $n = 11$, and 21 Chebyshev points

$$t_i \quad = \quad \cos(\tfrac{i\pi}{k}), \quad i = 0, 1, \ldots, k$$

in the interval $[-1, 1]$. You do the polynomial interpolation as we discussed in Homework 4, except that the Vandermonde matrix $A$ is square. You can solve the resulting system of linear equations $Ax = b$ using $x = A\backslash b$, or your LU factorization code with partial pivoting. Plot your polynomial interpolants, and the original function in the two cases, for each value of $n$.

2. [**Heath 7.7**] The *gamma function* is defined by

$$\Gamma(x) \quad = \quad \int_0^\infty t^{x-1} e^{-t}\, \mathrm{d}t, \quad x > 0$$

For an integer argument $n$, the gamma function has the value

$$\Gamma(n) \quad = \quad (n-1)!,$$

so interpolating the data points

| | | | | | |
|---|---|---|---|---|---|
| $t$ | 1 | 2 | 3 | 4 | 5 |
| $y$ | 1 | 1 | 2 | 6 | 24 |

should yield an approximation to the gamma function over the given range.

(a) Compute the polynomial of degree four that interpolates these five data points. Plot the resulting polynomial as well as the corresponding values given by the built-in MATLAB function *gamma* over the domain $[1, 5]$.

(b) Use the MATLAB cubic spline routine *spline* to interpolate the same data, and again plot the resulting curve along with the built-in *gamma* function.

(c) Which of the two interpolants is more accurate over most of the domain?.

(d) Which of the two interpolants is more accurate between 1 and 2?.

3. [**Heath : 9.1**] The populations of two species, a prey denoted by $y_1$, and a predator denoted by $y_2$, can be modeled by the following autonomous, nonlinear ODE due to Volterra and Lotka.

$$y' = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} y_1(\alpha_1 - \beta_1 y_2) \\ y_2(-\alpha_2 + \beta_2 y_1) \end{bmatrix}$$

The parameters $\alpha_1$, and $-\alpha_2$ are the natural parameters in isolation of prey and predators, respectively. The parameter $\alpha_1 > 0$ since the population of the prey grows if there were no predators (we are implicitly assuming that the food supply for the prey is abundant). On the other hand the parameter $-\alpha_2$ for the predator is negative, because the predator population would die out in the absence of any prey; the predators depend exclusively on the prey for their food. The parameters $\beta_1$, and $\beta_2$ determine the effect of interactions between the two populations, where we are assuming that the probability of interaction is proportional to the product of the populations. Let us consider the prey-predator dynamics for the time interval $[0, 25]$. The parameters values are $\alpha_1 = 1$, $\beta_1 = 0.1$, $\alpha_2 = 0.5$, $\beta_2 = 0.02$, and initial populations $y_1(0) = 100$, and $y_2(0) = 10$.

(a) Use the MATLAB ODE solver *ode45* to solve the Lotka-Volterra model with the above parameters over the time interval specified.

(b) Plot each of the two populations as a function of time, and on a separate graph plot he trajectory of the point $(y_1(t), y_2(t))$ in the plane as a function of time. The latter is sometimes called a *phase potrait*.

(c) Give a physical interpretation of the behavior you observe. Try other initial populations, and observe the results using the same type of graphs.

(d) Can you find nonzero initial populations such that either of the populations eventually becomes extinct?.

(e) Can you find nonzero initial populations that never change?.

(f) Repeat the above calculations, but this time use the *Leslie-Gower* model

$$\begin{aligned} y_1' &= y_1(\alpha_1 - \beta_1 y_2) \\ y_2' &= y_2(\alpha_2 - \beta_2 \tfrac{y_2}{y_1}) \end{aligned}$$

Use the same parameter values except take $\beta_2 = 10$. How do the behavior of the solutions differ between the two models?.

4. [**Heath : 9.6**] The following system of ODE's formulated by Lorenz, represents a crude model of atmospheric circulation.

$$\begin{aligned} y_1' &= \sigma(y_2 - y_1) \\ y_2' &= ry_1 - y_2 - y_1 y_3 \\ y_3' &= y_1 y_2 - b y_3 \end{aligned}$$

(a) Taking $\sigma = 10, b = \frac{8}{3}, r = 28$, and initial values $y_1(0) = y_3(0) = 0$, and $y_2(0) = 1$, solve this ODE from $t = 0$ to $t = 100$. Plot each of the $y_1$,$y_2$, and $y_3$ as a function of $t$, and also plot each of the trajectories $(y_1(t), y_2(t))$, $(y_1(t), y_3(t))$, and $(y_2(t), y_3(t))$ as a function of $t$, each on a separate plot.

(b) Try perturbing the initial values by a tiny amount, and see how much difference this makes in the final value of $y(100)$.