The Impact of Nonlinear Amplification on the Performance of CDMA Systems

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Abstract **- An analytical formulation separating the correlated and uncorrelated distortion spectrum when a wireless communication signal is processed by a nonlinear amplifier is presented. The correlated distortion spectrum relates directly to gain compression or expansion while the uncorrelated distortion spectrum contributes to degradation of the Signal-to- Noise Ratio (SNR). The uncorrelated distortion spectrum is used to calculate bit error rate (BER) when a nonlinear amplifier is followed by an AWGN channel. BER performance predicted from the uncorrelated distortion spectrum is in excellent agreement with simulated BER.**

***Index Terms*— Nonlinear power amplifier, in-band distortion, CDMA, BER, SNR, autocorrelation function, uncorrelated distortion noise.**

I. INTRODUCTION

One of the grand challenges of radio system design is to develop analysis techniques which directly relate system performance to circuit performance. In the case of digital mobile radio, trade-offs in power consumption and noise performance are possible when circuit performance criteria such as distortion, phase noise, noise figure, etc. are directly related to system measurements such as bit error rate (BER), symbol error rate (SER), or frame error rate (FER).

Nonlinear characteristics of RF and microwave amplifiers result in two main system impairments of communication signals that limit system performance. The first is co-channel distortion which is manifested as a degradation of SNR and ultimately as a degradation of BER. The second is spectral regrowth (SR) distortion which is responsible for degradation of SNR for users operating in the adjacent or alternate channels. Generally, systems with linear modulation schemes are more susceptible to power amplifier (PA) nonlinearity because of amplitude variations of the modulation envelope. Amplitude signal peaks are clipped when a nonlinear amplifier is operated near the saturated power level resulting in spectral regrowth and gain compression. Co-channel distortion and spectral regrowth originate from the same nonlinear phenomena and both are difficult to predict at the system and circuit design level.

The most common system level measure of distortion is the adjacent channel power ratio (ACPR) specification which compares the power level of the desired signal to the distortion power level in the adjacent channel [1]. The main system concern regarding ACPR is that distortion in the adjacent channel degrades the SNR of users operating in the adjacent channel. In contrast, a direct system specification for co-channel distortion does not exist because of the difficulty associated with directly measuring the co-channel distortion; however, system specifications for error vector magnitude (EVM) and rho, the cross-correlation coefficient, are measures of signal waveform quality which are directly related to SNR [2]. Ultimately, SNR is directly related to BER for a particular modulation scheme and thus it is desirable to predict the SNR degradation and BER that is directly related to the distortion generated when a signal is applied to a nonlinear circuit.

This paper presents an analytical method for calculating SNR of a signal that is applied to nonlinear circuit. The analysis is based on separation of the correlated and uncorrelated components of the distortion spectrum at the output of the nonlinear amplifier. The uncorrelated component of the power spectrum represents the in-band noise floor of the signal while the correlated component accounts for the gain compression and/or expansion characteristic. The ratio of desired output signal power to the uncorrelated co-channel distortion power is used to calculate SNR and BER for a CDMA IS-95 signal. The output power spectrum is analyzed from the output autocorrelation function of a signal applied to a complex power series behavioral model of a bandpass nonlinearity. The analytical model is verified against simulations where the forward link transmit/receive system with a nonlinear amplifier is simulated in MATLAB.

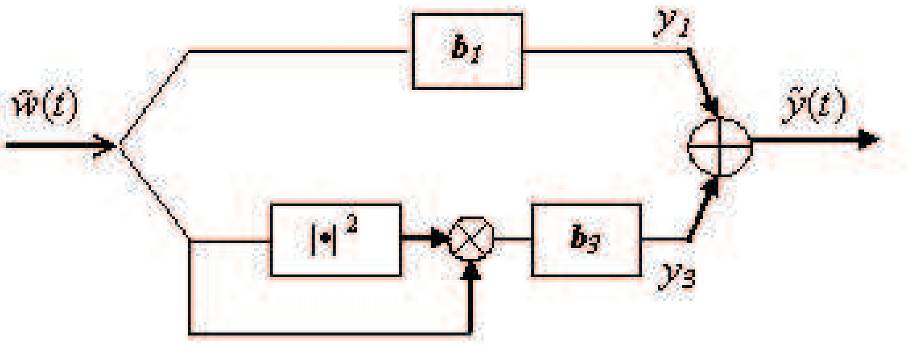

Fig. 1. Envelope behavioral model.

II. Nonlinear Spectral Analysis

In order to evaluate the co-channel distortion introduced by a memoryless nonlinearity, let us consider a memoryless model which is characterized by the envelope (power series) model shown in Fig. 1:

$$\tilde{y}(t) = \sum_{n=1}^{N} b_n \tilde{w}(t) |\tilde{w}(t)|^{n-1} \tag{1}$$

where $\tilde{w}(t)$ is the complex envelope of the input waveform and the coefficients b_n can be obtained by polynomial fitting of the measured AM-AM and AM-PM coefficients. The output autocorrelation function is defined as [3]

$$R_{\tilde{y}\tilde{y}}(\tau) = E[\tilde{y}(t)\tilde{y}^*(t+\tau)].$$

Now, let $\tilde{w}_1 = \tilde{w}(t)$ and $\tilde{w}_2 = \tilde{w}(t+\tau)$ and using (1), then the output autocorrelation function is

$$R_{\tilde{y}\tilde{y}}(\tau) = \sum_{n=1}^{N} b_n b_m^* R_{\tilde{w}^n \tilde{w}^m}(\tau)$$

where

$$R_{\tilde{w}^n \tilde{w}^m}(\tau) = E\left[\tilde{w}_1^{\frac{(n+1)}{2}} \tilde{w}_1^{*\frac{(n-1)}{2}} \tilde{w}_2^{\frac{(n-1)}{2}} \tilde{w}_2^{*\frac{(n+1)}{2}} \right]$$

and the output Power Spectral Density (PSD) can be developed from the Fourier transform of the autocorrelation function as [1]

$$S_{\tilde{y}\tilde{y}}(f) = \sum_{n=1}^{N} \sum_{m=1}^{N} b_n b_m^* S_{\tilde{w}^n \tilde{w}^m}(f)$$

where

$$S_{\tilde{w}^n \tilde{w}^m}(f) = \int_{-\infty}^{\infty} R_{\tilde{w}^n \tilde{w}^m}(\tau) e^{-j\omega\tau d\tau}$$

Therefore, using the signal model and the behavioral model coefficients, the output spectrum can be developed.

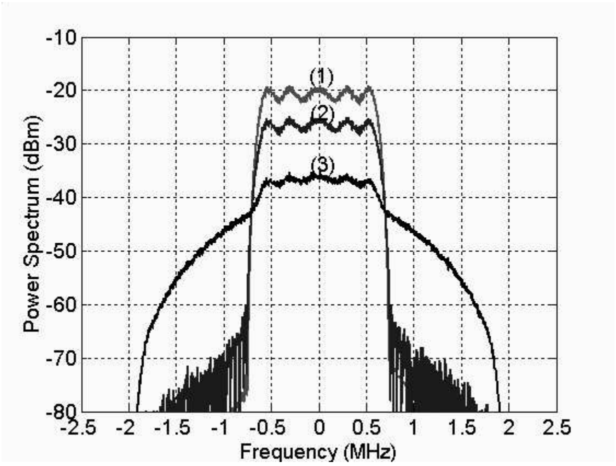

Fig. 2. Output spectrum partitioned into: (1) linear; (2) gain compression and (3) spectral regrowth.

Note that the output spectrum consists of a linear term, gain compression terms and spectral regrowth terms. Fig. 2 shows the output spectrum partitioned into linear, gain compression and spectral regrowth components. This partition however is not adequately useful for determining the output SNR since the spectral regrowth terms consist of correlated component which contribute to the linear output and an uncorrelated component which represent an additive noise. In the model described by (1), the linear and nonlinear branches may be statistically correlated. Therefore, we need to convert this model into a model with uncorrelated outputs as shown in Fig 3. We follow a procedure used in [4] for system identification to separate the correlated and uncorrelated components of the model output. Let α be a complex coefficient that represents the fraction of the cubic term that is correlated with the linear response, therefore for a third order nonlinearity we define

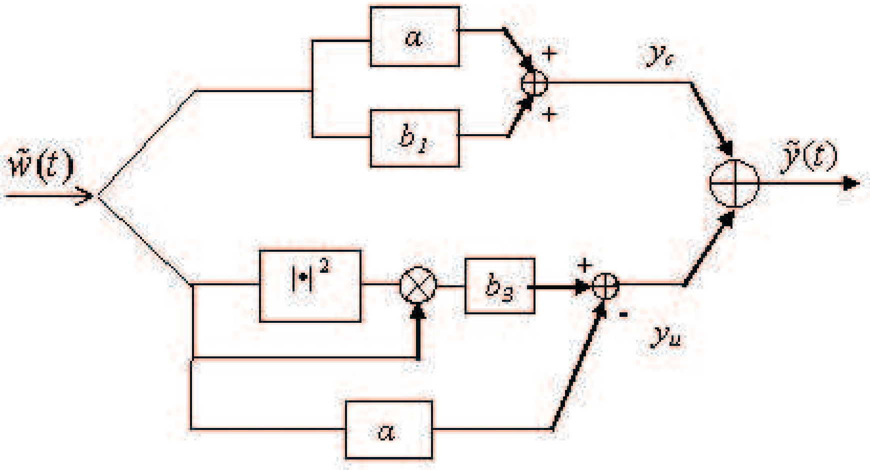

Fig. 3. Envelop nonlinear model with uncorrelated outputs.

a new set of outputs: $\tilde{y}_c(t) = b_1\tilde{w}(t) + \alpha\tilde{w}(t)$ which is correlated to the input signal and $\tilde{y}_u(t) = b_3\tilde{w}(t)|\tilde{w}(t)|^2 - \alpha\tilde{w}(t)$ which is uncorrelated to the input signal. The new outputs $y_c(t)$, $y_u(t)$ have zero cross correlation, therefore

$$E[\tilde{y}_c(t)\tilde{y}_u^*(t)] = 0.$$

It follows that

$$\alpha = \frac{b_3 R^*_{\tilde{w}^1\tilde{w}^3}(0)}{R^*_{\tilde{w}\tilde{w}}(0)}$$

and the output autocorrelation function can now be written as

$$R_{\tilde{y}\tilde{y}}(\tau) = R_{\tilde{y}_c\tilde{y}_c}(\tau) + R_{\tilde{y}_u\tilde{y}_u}(\tau)$$

where

$$R_{\tilde{y}_c\tilde{y}_c}(\tau) = |\alpha + b_1|^2 R_{ww}(\tau)$$

and

$$R_{\tilde{y}_u\tilde{y}_u}(\tau) = |b_3|^2 R_{\tilde{w}^3\tilde{w}^3}(\tau) - |\alpha|^2 R_{ww}(\tau).$$

In order to develop the relationship between the co-channel distortion and system BER, it is useful to express the above formulation in the frequency domain, theretofore

$$\alpha = \frac{b_3 \int_{-\infty}^{\infty} S^*_{\tilde{w}^1\tilde{w}^3}(f)df}{\int_{\infty}^{\infty} S^*_{\tilde{w}\tilde{w}}(f)df}$$

and it follows that the output spectrum is expressed as

$$S_{\tilde{y}\tilde{y}}(f) = S_{\tilde{y}_c\tilde{y}_c}(f) + S_{\tilde{y}_u\tilde{y}_u}(f)$$

where

$$S_{\tilde{y}_c\tilde{y}_c}(f) = |b_1 + \alpha|^2 S_{\tilde{w}\tilde{w}}(f)$$

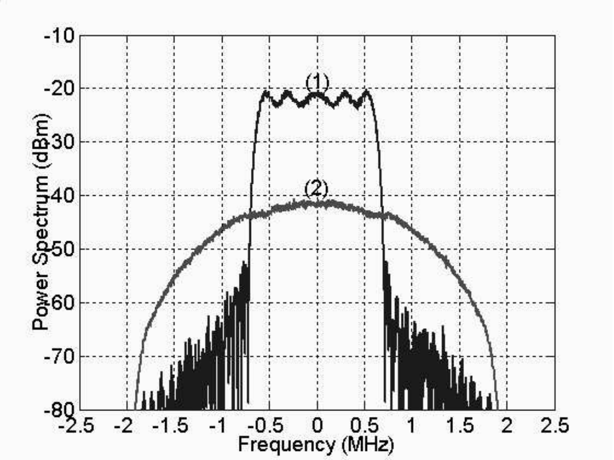

Fig. 4. Output spectrum partitioned into: (1) correlated; and (2) uncorrelated components.

and

$$S_{\tilde{y}_u\tilde{y}_u}(f) = |b_3|^2 S_{\tilde{w}^3\tilde{w}^3}(f) - |\alpha|^2 S_{\tilde{w}\tilde{w}}(f).$$

Fig. 4 shows the output spectrum partitioned into correlated and uncorrelated components. Note that this formulation enables the uncorrelated output distortion to be treated as an additive noise similar to the AWGN and enables the actual SNR and hence the BER to be determined since the actual noise-like component of the output nonlinear power is determined. Therefore, the actual received signal consists of a correlated component $\tilde{y}_c(t)$ which contributes to the gain compression/expansion, an uncorrelated distortion component $\tilde{y}_u(t)$ and an AWGN component $\tilde{n}(t)$ which represents the channel noise:

$$\tilde{r}(t) = \tilde{y_c}(t) + \tilde{y}_u(t) + \tilde{n}(t) \tag{2}$$

and the effective SNR including distortion and channel additive noise is defined as

$$\mathrm{SNR} = \frac{\int_{-B/2}^{B/2} S_{\tilde{y}_c\tilde{y}_c}(f)df}{\int_{-B/2}^{B/2} S_{\tilde{y}_u\tilde{y}_u}(f)df + N_0 B}$$

where $N_o/2$ is the power spectral density of an AWGN component. The probability of bit error can then be evaluated, for example, from the error probability of QPSK or BPSK system as [5]

$$P_e = Q\left(\sqrt{2\mathrm{SNR}}\right)$$

Note that SNR is a function of both the nonlinear distortion and the energy per bit-to-AWGN ratio E_b/N_0.

III. Simulation Results

The analytical evaluation of BER using the SNR obtained from the output spectrum was verified by simulations. An IS-95 forward link signal was generated and then applied to the nonlinear model using coefficients extracted using the AM-AM and AM-PM measured characteristics. AWGN was then added to the distorted signals and the resulting signal was then demodulated and decoded. The input power of the signal was swept and the number of erroneous bits were counted at each power level and E_b/N_0. The simulations were repeated to account for the different combinations of consecutive bits. Fig. 5 shows the probability of error as a function of E_b/N_0 and at different power levels using the analytical formula of BER. Note that as co-channel distortion increases, the effective SNR is degraded and this is manifested as an increase in the BER. The analytical model shows a good agreement with simulated values.

IV. Conclusion

We have developed a nonlinear spectral analysis to estimate the effective co-channel distortion introduced by power amplifier nonlinearity. The analytical formulation presented here enables the probability of error to be directly related to nonlinear distortion. The uncorrelated co-channel distortion is obtained from the output spectrum estimated using signal realizations and measured AM-AM and AM-PM characteristics.

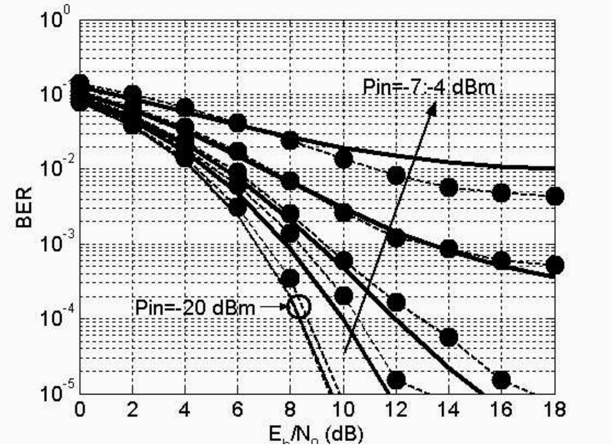

Fig. 5. Output BER vs. E_b/N_0 for different input power levels; solid: analytic; and ●: simulation.

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