

The Importance of Nonlinear Order in Modeling Intermodulation Distortion and Spectral Regrowth

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Abstract- Modeling of distortion in nonlinear RF and microwave amplifiers requires high order nonlinear modeling to capture behavior with digitally modulated signals but only low order modeling to capture the response with discrete spectra as in two-tone testing. This paper establishes model order requirements for successful behavioral modeling for system explorations.

I. INTRODUCTION

Intermodulation Distortion Ratio (IMR) is an adequate performance measure for the nonlinear behavior of RF amplifiers with analog modulated signals such as FM. This is because these signals can be represented as a number of discrete tones and so two-tone testing accurately captures nonlinear behavior. However, relating two-tone properties to spectral distortion with digitally modulated signals has proved elusive. With digitally modulated signals, the level of spectral regrowth of the spectrum in neighboring channels is a fundamental performance parameter with distortion quantified by the Adjacent Channel Power Ratio (ACPR). A relationship between IMR of discrete tone signals and Spectral Regrowth (SR) of digitally modulated signals has long been supposed as they are both due to the same nonlinear processes. Generally, it is assumed that a device that has better IMR should have better ACPR. However, several experimental investigations of IMR and SR for various combinations of devices and types of signals (discrete tone and two digitally modulated formats) [1, 2] have generally indicated little apparent relationship. Significant analytic work has also been done to investigate the relationship between intermodulation levels and ACPR [3, 4, 5]. The major problem with all of these analyses is the relatively low order of the nonlinearity that was considered (usually limited to third order) and not considering the dependence of ACPR on the modulation and encoding schemes.

It is the purpose of this paper to explore possible relationships and to show that lower order nonlinear modeling can be used to capture discrete tone distortion but high order modeling is important for capturing the response with digitally modulated signals. This is because the nonlinear amplifier is then presenting its most extreme nonlinearity at the peaks of the signals. Therefore the relationship between the two mentioned measures of nonlinearity cannot be established specifically for high drive levels. We demonstrate that higher order nonlinear modeling must be used in the behavioral modeling of RF systems with digitally modulated signals than in the behavioral modeling of two-tone response.

II. ANALOG SYSTEM DISTORTION

The nonlinear performance of an amplifier with analog signals is generally quantified using two-tone testing. The troublesome tones at the output of the amplifier are commonly called the third order intermodulation components IM_3 . This terminology is loose as other intermodulation orders are involved. Intermodulation Ratio (IMR_3) is determined by considering the response of two equal amplitude input tones, of frequencies f_1 and f_2 . IMR_3 is the ratio of the power of the lower IM_{3L} (or upper, IM_{3U}) intermodulation tone, here at $f_3 = 2f_1 - f_2$ (or $f_4 = 2f_2 - f_1$) to the power in one of the fundamental tones (f_1 and f_2):

$$IMR_{3L} = \frac{P_{IM_{3L}}}{P_{Fund}} = \frac{P_{2f_1 - f_2}}{P_{f_1}}$$

and IMR_{3U} is defined in the same way.

Generalized power series analysis [6] can be used to model intermodulation distortion of a two-tone signal. The output at a particular frequency ($= n_1 f_1 + n_2 f_2$) is then the vector addition of a number of intermodulation products IP's. For IMR_{3L}

and the behavioral model there is only one IP: $n_1 = 2$, $n_2 = -1$, and so:

$$IM_{3L} = K[1+T'] \quad (1)$$

where:

$K = 2a_3 \frac{3!}{2^3} \frac{X_1^2}{2} X_2^*$ is the intermodulation term, * indicates complex conjugation, X_1 and X_2 are the phasors of the tones at f_1 and f_2 , the a_i is the i th order term of the behavioral model which represents a memoryless nonlinear system as a complex power series. The saturation term is:

$$T' = \sum_{\alpha=1}^{\infty} \left\{ \left(\frac{(3+2\alpha)!}{3! 2^{2\alpha}} \right) \frac{a_{3+2\alpha}}{a_3} \right\} \sum_{\substack{S_1, S_2 \\ S_1+S_2=\alpha}} \Phi'$$

where

$$\Phi' = \frac{|X_1|^{2S_1} 2! |X_2|^{2S_2}}{S_1!(2+S_1)! S_2!(1+S_2)!}$$

The intermodulation term K , has a simple relationship to the input tones and varies as the cube of the level of one of the input tones yielding the classic 3:1 slope. However, this is valid only when the saturation term T' is zero (for small signals) but as the signal levels become larger, this term grows because of the contribution of the fifth and higher order components.

In the following, we discuss traditional measures of nonlinearity and their limitations.

A) Intercept Points

Traditionally, the third order output intercept point (IP_3) describes nonlinearity of a power amplifier as it is a quantity specified by the manufacturer. By definition, the i th order input intercept point IIP_i is the intercept point of extrapolated output power-input power curve and extrapolated P_{IMi} -input power curve, where P_{IMi} denotes the i th order intermodulation distortion power calculated from a two tone (separated by small frequency difference) test of the nonlinear device. The intercept points represent a good description of amplifier nonlinearity because they are independent of the input and output powers and are characteristics of the amplifier alone. However, IIP_i is an extrapolated value and indicate the nonlinear response only in the region where the P_{IMi} has a slope equal to i . The calculated IIP_i is not a reliable indicator of performance outside that region and it cannot predict IMR_i over the entire power

range. Therefore, IIP_i may not be the proper parameter to characterize nonlinearity with non-constant envelope modulated signal. In addition, an intercept point of order i is constant only when that particular i th intermodulation order is present alone. Since single order nonlinearity in real nonlinear systems is rarely present, the intercept point concept does not accurately represent nonlinearity.

B) 1 dB Compression Point

A very common parameter of nonlinearity is the 1 dB compression point. The 1dB compression point is the input power at which the extrapolated linear response is greater than that power by 1 dB.

Using the model in [7], the output of the nonlinear device for a single tone input is expressed as:

$$Y = a_1 X + \sum_{\alpha=1}^{\infty} \left(\frac{(1+2\alpha)!}{2^{2\alpha} \alpha! (1+\alpha)!} \right) a_{1+2\alpha} |X|^{2\alpha} X$$

$$\text{i.e.: } Y = a_1 X + \frac{3}{4} a_3 X^3 + \frac{5}{8} a_5 X^5 + \frac{35}{64} a_7 X^7 + \dots$$

A 1 dB compression point correspond to a factor of $10^{-1/20} = .89$ change in voltage gain. Therefore, the corresponding value of X where the power drops to 1 dB below the linear gain is found by solving :

$$a_1 X + \frac{3}{4} a_3 X^3 + \frac{5}{8} a_5 X^5 + \frac{35}{64} a_7 X^7 + \dots = .89 a_1 X \quad (2)$$

Note that the value of the 1 dB compression point is also dependent on other intermodulation product above 3.

The relation between the 1 dB compression point and the third order intercept point can be understood by considering a pure third order nonlinearity where the value of X in (2) (neglecting terms of order >3) can be expressed as:

$$X_{1dB} = \sqrt{.11(4/3)} |a_1 / a_3|$$

and the value of X at the intercept point is:

$$X_{IIP_3} = \sqrt{(4/3)} |a_1 / a_3|$$

therefore, $IIP_3 - P_{1dB}$ (= 9.6 dB) is a fundamental lower limit for the difference. This difference becomes higher when higher order terms are considered because of the decrease in P_{1dB} as the influence of fifth and higher order intermodulation products is becoming more significant. This imposes

a restriction on the applicability of IIP_3 as measure of nonlinearity, where it becomes more applicable when P_{1dB} is higher.

III. DIGITAL SIGNAL DISTORTION

Interpretation of the digital process with a digitally modulated signal is complicated by the input signal being best represented by a waveform and the output signal interpreted in the frequency domain. Such signals are most conveniently represented by the autocorrelation function because of their random nature. In [8], a generalized autocorrelation function at the output of the nonlinear system was derived. Assuming that $z(t)$ is a wide sense stationary process that represent the complex envelope of a CDMA signal, the autocorrelation of the signal at the output of a nonlinear amplifier can be written as:

$$\tilde{R}_{gg}(\tau) = \sum_{m=1}^N \sum_{n=1}^N \frac{a_n a_m^*}{2^{n+m-2}} \binom{n}{n+1} \binom{m}{m+1} R_{z_n z_m}(\tau) \quad (3)$$

Where $R_{z_n z_m}(\tau) = E[z_1^{\frac{n+1}{2}} z_1^{*\frac{n-1}{2}} z_2^{\frac{m-1}{2}} z_2^{*\frac{m+1}{2}}]$, E is the expected value and a_i is the i -th order term of the behavioral model. The output power spectrum is obtained from the Fourier transform of (3):

$$S_{gg}(f) = \sum_{m=1}^N \sum_{n=1}^N \frac{a_n a_m^*}{2^{n+m-2}} \binom{n}{n+1} \binom{m}{m+1} S_{nm}(f) \quad (4)$$

where: $S_{nm}(f) = \int_{-\infty}^{\infty} R_{z_n z_m}(\tau) e^{-j\omega\tau} d\tau$.

Therefore, the output spectrum is a sum of the Fourier transform from each component of the autocorrelation function weighted by the appropriate power series coefficient. ACPR is defined as the ratio of the power that lies out of band to the power in the main channel. In the IS-95 Standard, ACPR is defined as the ratio of the adjacent channel power in a 30kHz resolution bandwidth swept over the adjacent channel, to the total power in the main channel (1.23 MHz Band Width).

IV. SIMULATION RESULTS

The intermodulation and the statistical analyses in section II and III were used to simulate distortion in both analog and digital systems. The amplifier considered here is the GaAs MESFET 900 MHz driver amplifier described in [8], where a complex

power series of order 13 was fitted to the AM-AM and AM-PM measured data.

A) Intermodulation Distortion:

Fig. 1, shows the third order intermodulation ratio IMR_3 vs. output power for a two tone input signal around 2 GHz. Initially, the IMR_3 is a third order process depending only on the third order power series term. As the level of input tones increase, the higher order terms of the behavioral model becomes important and the P_{IM3} vs. P_{in} curve does not have a constant slope of 3 as expected. The slope grows above 3 after a certain limit. This is seen with the measured and calculated IMR_3 . The measured IMR_{3L} and IMR_{3U} are almost identical. Fig 2 shows the output power vs. input power for different maximum orders of nonlinearity. Table 1 shows the P_{1dB} and the difference between IIP_3 and P_{1dB} for different maximum orders of nonlinearity. In all the simulations, it can be seen that considering the fifth order term is important in capturing the response up to medium to large power levels. However, little is gained by considering a higher order behavioral model.

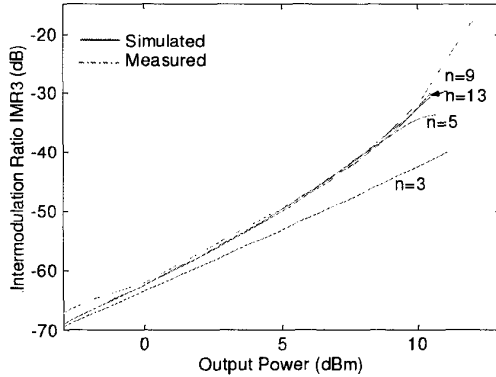


Fig. 1 Intermodulation Ratio (IMR_3) for different maximum order of nonlinearity.

Table 1: P_{1dB} and $IIP_3 - P_{1dB}$ for different maximum order of nonlinearity ($IIP_3 = 4.5$ dBm).

Order	P_{1dB}	$IIP_3 - P_{1dB}$
3	-5.1	9.6
5	-8.3	12.8
7	-8.4	12.9
13	-8.4	12.9

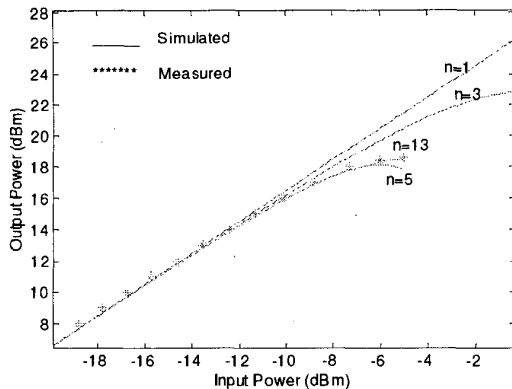


Fig. 2. Input-Output power for different maximum order of nonlinearity.

B) Adjacent Channel Power Ratio

The response of the amplifier considered previously was simulated with a CDMA signal at 900 MHz. The measured and calculated results for ACPR using different maximum orders of the behavioral model are shown in Fig. 4. Low order behavioral models, predict the proper trend in the ACPR but the agreement with measurements is good only when very high order (11 th or 13 th) behavioral models are used.

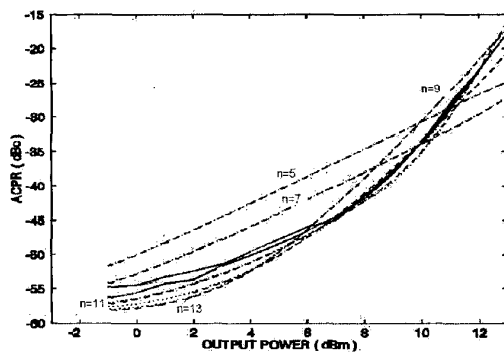


Fig. 4 Measured and simulated ACPR for different maximum order of nonlinearity (dashed: simulated, solid: measured).

V. CONCLUSION

We have shown that the third order assumption in modeling power amplifiers is not adequate to capture the distortion introduced by the nonlinear behavior and therefore, a relationship between IMR and ACPR cannot be established adequately. The simulations

showed that for analog systems, considering fifth order of nonlinearity is enough to give good estimates of IMR while eleventh order is necessary to estimate the ACPR in a digital RF system.

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