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# Linear amplification by time-multiplexed spectrum

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**Abstract:** A new waveform processing technique for reducing intermodulation distortion (IMD) generated by an amplitude-modulated signal is presented. The technique eliminates amplifier-generated IMD by time-multiplexing portions of the signal spectrum prior to amplification. The input signal is described as a sum of sinusoidal signals. The peak-to-average ratio of the time-multiplexed amplifier input is lower than that of its non-multiplexed counterpart, but it contains spectral aliases at multiples of the switching frequency. Thus, the reduction in distortion of the desired amplified signal is achieved at the expense of momentarily widening the signal bandwidth. Following amplification, the desired signal is recovered using a bandpass filter which spreads the signal in time and narrows its bandwidth. An analytical expression for distortion reduction when amplifying two multiplexed carriers is developed along with measurements verifying the theory. Distortion reduction is demonstrated experimentally at 3.6 GHz for 2–20 multiplexed carriers, and linear recovery is demonstrated for four multiplexed carriers. The technique reduces third-order IMD by 8–22 dB in experimental measurements of three different amplifiers. The presented technique has the unique property of improving linearity without requiring feedback, feedforward cancellation or calibration.

## 1 Introduction

Linear amplification by time-multiplexed spectrum (LITMUS) is a new time–frequency waveform processing technique to reduce intermodulation distortion (IMD) associated with envelope amplitude modulation. Since a narrow-band digitally modulated signal can be represented by amplitude and phase components, and since an amplitude-modulated signal can be represented as a sum of simultaneous sinusoids [1], a narrow-band digitally modulated signal containing amplitude modulation can be reconstructed from a sequence of short sinusoidal pulses, each of which contains the requisite amplitude and phase information. For the LITMUS technique, the duration of each pulse is proportional to its frequency's component amplitude and the phase of each pulse is proportional to its component phase. Time-separated components are amplified in succession and then recombined to produce an amplified replica of a composite signal. In this way, LITMUS enables digitally modulated signals with high peak-to-average ratios to be amplified with the efficiency often considered only possible with constant-amplitude signals.

The concept behind LITMUS is explained using a frequency-domain representation of a digitally modulated signal. A modulated signal is described as a finite sum of sinusoids representing the Fourier coefficients of the signal [2]. Intermodulation and harmonic distortion result when amplitude modulation, created by the sum of sinusoids, interacts with the non-linear characteristic of a transmitter. LITMUS prevents interaction between the amplitude modulation and transmitter non-linearity by applying only one sinusoid at a time to the non-linear circuit. The spectral components are multiplexed in time, using natural sampling, at a rate much faster than the symbol rate of the information signal. The desired amplified signal is completely recovered at the output using a bandpass filter to remove sampling aliases generated during the time-multiplexing process. Theoretically, LITMUS completely suppresses IMD about the desired signal when ideal multiplexing is used; however, practical switching bandwidths limit experimentally-observed third-order intermodulation distortion (IM3) suppression to 8.8–22.7 dB depending on the bandwidth available for the time-multiplexed signal. This paper presents the theory behind LITMUS and experimental results verifying the theory. To the authors' knowledge, this work presents the first theoretical and experimental analysis of IMD generated by time-multiplexed signals.

Relevant transmitter linearisation techniques are reviewed and contrasted to the LITMUS concept in Section 2. The theoretical framework for the technique and the IMD response for a time-multiplexed two-tone signal are presented in Section 3. An extension of this theory to four tones is given in the Appendix. Measurements that verify the theoretically predicted distortion reduction are presented in Section 4. Time-domain captures and distortion measurements that verify the non-multiplexed, linearised signal recovery are given in Section 5.

## 2 Background

The demand for linear and power-efficient transmitters continues to grow as wireless systems migrate towards broadband data and multimedia services. New systems adaptively employ a diverse set of spectrally efficient modulation schemes ranging from binary phase shift keying (BPSK) to orthogonal frequency division multiplexed (OFDM) with 64-QAM (quadrature amplitude modulation) symbols on each subcarrier. As a result, transmitter specifications for linearity and signal quality are increasingly stringent to accommodate the higher signal-to-noise ratio (SNR) and adjacent channel interference requirements. Options for linear transmitter designs include the use of average power backoff with linear transmitters, or employing linearisation techniques to achieve the spectral emission requirements. Linearisation techniques include predistortion correction, cartesian feedback correction or feedforward distortion cancellation [3]. These techniques correct the distorted transmitter signal by either predistorting the input signal [4, 5] or by cancelling the distortion at the output of the transmitter [6]. In either case, knowledge of the non-linearity being corrected is required either in the form of a look-up table, trained model or feedback circuit; or cancellation calibration is necessary to achieve improvement in linearity.

LITMUS is a signal-processing-based linearisation technique with the unique property of improving linearity of the desired signal without requiring knowledge of the non-linear circuit or calibration of a cancellation network. The signal is encoded using time-multiplexed sinusoids such that the generation of IMD is prevented by applying only one sinusoid at a time to the non-linear circuit. Ideal switch multiplexing of sinusoids completely prevents generation of intermodulation; however, bandwidth and switching limitations result in the generation of finite distortion at frequencies around the desired signal. Reduction of IMD using time-multiplexed carriers was demonstrated experimentally for an optical cable TV (CATV) system [7], and more recently for a very high frequency (VHF) amplifier [8]. The work presented here extends the earlier experimental results by providing the theory of distortion reduction for time-multiplexed sinusoids along with the measurements demonstrating consistent distortion reduction for up to 20 arbitrarily phased multiplexed tones.

## 3 Distortion reduction theory

The theory of LITMUS is described here, beginning with an analysis of time-multiplexed sinusoids passed through a memoryless non-linearity. The analysis begins using a time-multiplexed two-tone signal and illustrates the principals behind LITMUS. Extensions to higher numbers of tones are illustrated experimentally. The analysis contrasts distortion reduction using time-multiplexed sinusoids and using cancellation linearisation. With cancellation schemes, distortion is reduced through the generation of intermodulation components that cancel the IMD resulting from non-linear amplification. In contrast, time-multiplexed sinusoids do not correct for specific non-linear characteristics; rather, IMD generation is suppressed as a result of the technique regardless of the specifics of the non-linear circuit.

### 3.1 Time-multiplexed sinusoids

The theory of time-multiplexed signals [1] is used to establish a description of a sum of time-multiplexed sinusoidal signals. Let the input to the amplifier of Fig. 1 be the switched-tone (time-multiplexed sinusoidal) signal given by

$$v_i(t) = \sum_{n=1}^{N} A_n \cos(\omega_n t + \phi_n) s\left(t - \frac{n-1}{N} T_s\right) \qquad (1)$$

where $N$ is the number of tones, $A_n$ are the amplitudes of each tone, $\omega_n$ are the frequencies of each tone, $\phi_n$ are the initial phases of each tone and $s(t)$ is the periodic switch waveform given by

$$s(t) = u(t) - u(t - T_s/N) = s(t + T_s) \qquad (2)$$

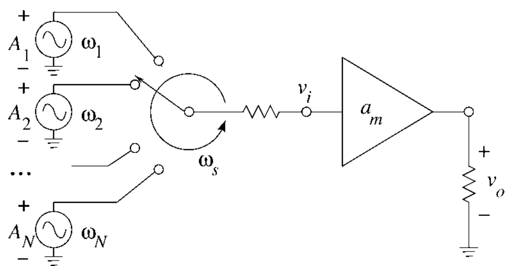


**Figure 1** *LITMUS circuit architecture*

Ideally, only one $\omega_1, \ldots, \omega_N$ tone is applied to the non-linear amplifier at any one time where $T$ is the switching period. The Fourier transform of $s(t)$ is

$$S(\omega) = \sum_{k=-\infty}^{+\infty} \frac{2\sin(k\pi/N)}{k} \delta(\omega - k\omega_s) \qquad (3)$$

where $\omega_s = 2\pi/T_s$ is the switching frequency and each $\pm k$ value is a harmonic of this frequency. Including only $k = -1$, 0, 1 in this expansion produces sinusoidal switching at $\omega_s$; more ideal switching is achieved with higher values of $k$.

The frequency-domain representation of $v_\mathrm{i}(t)$ equals the convolution of this switching signal with the fundamental tone set

$$V_\mathrm{i}(\omega) = \frac{1}{2\pi} \sum_{n=1}^{N} \left\{ \begin{array}{l} A_n e^{j\phi_n} [\pi\delta(\omega - \omega_n) + \pi\delta(\omega + \omega_n)] \\ \quad * S(\omega) e^{-j\omega T_s [n-1]/N} \end{array} \right\} \qquad (4)$$

Performing the convolution gives

$$V_\mathrm{i}(\omega) = \sum_{n=1}^{N} A_n \sum_{k=-\infty}^{+\infty} \times \left\{ \begin{array}{l} \dfrac{\sin(k\pi/N)}{k} e^{-j\omega T_s (n-1)/N} \\ \quad \times [e^{j\phi_n} \delta(\omega - k\omega_s - \omega_n) \\ \quad + e^{-j\phi_n} \delta(\omega - k\omega_s + \omega_n)] \end{array} \right\} \qquad (5)$$

An example, switched-tone spectrum for $N = 10$ is given in Fig. 2. When transformed back into the time domain, $V_\mathrm{i}(\omega)$ becomes

$$v_\mathrm{i}(t) = \sum_{k=-\infty}^{+\infty} \frac{B_k}{\pi} \sum_{n=1}^{N} A_n \cos\left[(\omega_n + k\omega_s)\left(t - \frac{n-1}{N} T_s\right) + \phi_n\right] \qquad (6)$$

where $B_k = \sin(k\pi/N)/k$. The value of the sinc function at $k = 0$ is $\pi/N$. The time-multiplexed signal consists of the desired multitone signal weighted by $1/N$, plus a sum of aliased versions of the multitone signal centred at multiples of the switching frequency offset to the centre frequency. Before amplification, the peak-to-amplitude ratio of the multitone signal is reduced at the expense of momentarily widening its bandwidth by $k$ switching harmonics above and below the fundamental tones.

## 3.2 Intermodulation cancellation

In order to compare the linearity of an amplified switched-tone input to an amplified steady-state multitone signal with equal power in the fundamental tones, $v_i(t)$ from (6) is multiplied by $N$ prior to the following analysis.

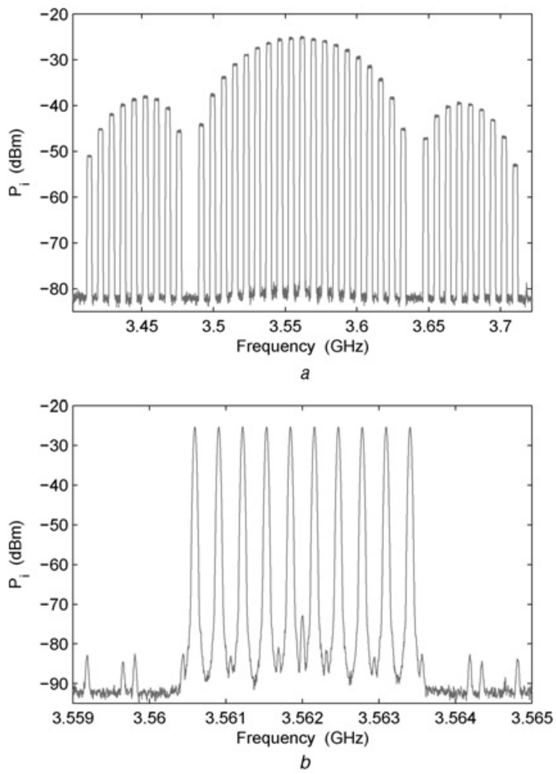


**Figure 2** *Example switched-tone spectrum from (5) as generated by the Polyphase Microwave QM3337A modulator, amplified by the MiniCircuits ZHL-1042J amplifier, and recorded by the Agilent E4445A spectrum analyser, $N = 10$, $P_i = -25.4$ dBm/tone*

*a* Wideband view, sinc envelope evident
*b* Narrowband view, individual tones evident

Assume that the output voltage of a memoryless amplifier may be written as

$$v_\mathrm{o}(t) = \sum_{m=1}^{M} a_m v_\mathrm{i}^m(t) \qquad (7)$$

where $a_m$ are the coefficients of the complex power series describing the amplifier response. To keep the expansions that follow tractable, let $C_N$ represent the inner summation of (6)

$$C_N = \sum_{n=1}^{N} A_n \cos\left[(\omega_n + k\omega_s)\left(t - \frac{n-1}{N} T_s\right) + \phi_n\right] \qquad (8)$$

The frequency terms of interest generated by (7) are those which fall closest to the fundamental tones $\omega_1, \ldots, \omega_N$. Assuming $\omega_s \gg \omega_N - \omega_1$ (i.e. that the time-multiplexing rate is chosen to be much greater than the signal bandwidth), the terms closest to $\omega_1, \ldots, \omega_N$ are the result



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of a non-zero third-order coefficient $a_3$

$$a_3 v_i^3(t) = a_3 \left\{ \sum_{k=-\infty}^{+\infty} \frac{B_k}{\pi} C_N \right\}^3$$
$$= a_3 \left\{ \sum_{\alpha=-\infty}^{+\infty} \frac{B_\alpha}{\pi} C_N \right\} \left\{ \sum_{\beta=-\infty}^{+\infty} \frac{B_\beta}{\pi} C_N \right\} \left\{ \sum_{\delta=-\infty}^{+\infty} \frac{B_\delta}{\pi} C_N \right\} \quad (9)$$

When cubing (6), $k$ must be split into three different indices – $\alpha$, $\beta$, and $\delta$ – to account for all unequal-index permutations that are possible when multiplying three sums.

Consider $N = 2$. If the bandwidth of the amplifier is such that it encapsulates only the fundamental tones ($k = 0$), then the third-order intermodulation products (i.e. the subset of tones from the triple-product that fall closest to $\omega_1$ and $\omega_2$) are

$$v_{\mathrm{IM3}}(t) = \frac{3}{4} a_3 \left\{ \begin{array}{l} A_1^2 A_2 \cos[(2\omega_1 - \omega_2)t + \omega_2 T_s/2 + 2\phi_1 - \phi_2] \\ + A_1 A_2^2 \cos[(2\omega_2 - \omega_1)t - \omega_2 T_s + 2\phi_2 - \phi_1] \end{array} \right\} \quad (10)$$

The 3/4 term is the classical result obtained in two-tone steady-state intermodulation analysis. If, in addition to the fundamental tones, the first set of switching harmonics $k = \pm 1$ is passed by the amplifier, additional terms are generated at integer multiples of the switching frequency $\omega_s$ away from $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$. Thus the terms around $2\omega_1 - \omega_2$ are at the frequencies

$$\omega_{\alpha,\beta,\delta} = (2\omega_1 - \omega_2) + (\alpha - \beta - \delta)\omega_s \quad (11)$$

where $\alpha$, $\beta$ and $\delta$ are interchangeable. In any case, the sum of two indices is subtracted from the third. The combinations of $\alpha$, $\beta$ and $\delta$ which produce spectral content at exactly $2\omega_1 - \omega_2$ for $k = -1, 0, 1$ are listed in Table 1. The '$V_{\mathrm{IM3}}$' row is the amplitude of each term normalised by $a_3 A_1^2 A_2$. The table for IMD products generated at $2\omega_2 - \omega_1$ is the same, except $V_{\mathrm{IM3}}$ is normalised by $a_3 A_1 A_2^2$ instead of $a_3 A_1^2 A_2$.

Some of the switching harmonics generate IM3 products which add to the non-multiplexed distortion (e.g. $\alpha = 0$, $\beta = -1$, $\delta = +1$), whereas others are $\pm\pi$ out-of-phase with the non-multiplexed products and subtract from the total distortion (e.g. $\alpha = +1$, $\beta = 0$, $\delta = +1$). If the amplifier passes the switching harmonics $k = \pm 1$ along with the fundamental tones at $k = 0$, the (normalised) amplitude of the third-order intermodulation is the sum of the last row of Table 1

$$V_{\mathrm{IM3}} = \frac{3}{4} + 2\frac{1}{\pi^2} - 4\frac{2}{\pi^2} = \frac{3}{4}\left(1 - \frac{8}{\pi^2}\right) \quad (12)$$

which is a 14.5-dB reduction in IMD from $k = 0$ only (i.e. no switching).

With higher-order switching harmonics ($|k| \geq 2$), the number of $\alpha$–$\beta$–$\delta$ combinations that add to the non-multiplexed distortion increases, but the number of combinations that are $\pm\pi$ out-of-phase with the distortion also increases. The subtraction always outweighs the addition while each converges to zero. The general result, as the amplifier passes the switching harmonics from $-\eta$ to $+\eta$ is

$$V_{\mathrm{IM3}} = \left[ \frac{3}{4} - \frac{9}{2\pi^2} \sum_{k=1}^{\eta} \frac{1}{k^2} \right] a_3 A^3 \quad (13)$$

whose limit, as $\eta \to \infty$ is

$$\lim_{\eta\to\infty} (V_{\mathrm{IM3}}) = \left[ \frac{3}{4} - \frac{9}{2\pi^2}\left(\frac{\pi^2}{6}\right) \right] a_3 A^3 = 0 \quad (14)$$

(see the Appendix for the derivations of (13) and (14)). This important result states that the theoretical IM3 at the output of the amplifier, as the circuit passes all switching harmonics of the fundamental two tones, is zero. It can also be shown that cancellation of IMD higher than third-order occurs by the same mechanism.

The intuitive idea that IMD is not generated, because only one tone at a time is presented to the non-linearity, is shown to be true when using ideal switching with infinite bandwidth to pass all harmonics. An extension of this theory to four tones is given in the Appendix. This theory may be extended to an arbitrary number of tones.

**Table 1** Time-multiplexed IM3, $N = 2$ and $k = -1, 0, 1$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\alpha$ | 0 | 0 | 0 | −1 | −1 | +1 | +1 |
| $\beta$ | 0 | −1 | +1 | −1 | 0 | 0 | +1 |
| $\delta$ | 0 | +1 | −1 | 0 | −1 | +1 | 0 |
| $V_{\mathrm{IM3}}$ | $\frac{3}{4}$ | $\frac{+1}{\pi^2}$ | $\frac{+1}{\pi^2}$ | $\frac{-2}{\pi^2}$ | $\frac{-2}{\pi^2}$ | $\frac{-2}{\pi^2}$ | $\frac{-2}{\pi^2}$ |

## 4 Experimental validation

In this section, measurements are taken to validate the theory presented in Section 3. Distortion reduction is demonstrated experimentally for two tones, over a wide range of switching harmonics.

The characteristics of microwave communication signals such as wideband code-division multiple access (W-CDMA) can be captured with higher numbers of uniform-amplitude arbitrarily phased multisines. Thus, additional measurements are taken using for a 3-MHz-bandwidth signal: (i) using four tones with arbitrary phases, over a range of power levels, and (ii) using up to 20 tones. These measurements demonstrate the applicability of LITMUS to practical communication scenarios.

## 4.1 Narrowband measurements, $N = 2$

A measurement system that demonstrates the distortion cancellation for a two-tone signal, that is $N = 2$, is given in Fig. 3. The signal generator is the Agilent E8267C. The two tones are centred on $f_0 = 465$ MHz, spaced 100 kHz apart and time-multiplexed with a switching frequency of $f_s = 6$ MHz. The amplifier is the Amplifier Research (AR) model 10W100C. A 20 dB coupler is used to tap off a portion of the amplifier's output so as not to damage the spectrum analyser, an Agilent E4445A. The results of this test are listed in Table 2.

The Agilent E8267C arbitrary waveform generator implements software filtering which is used to control the number of switching harmonics applied to the amplifier. First, only the fundamental two tones are applied to the AR10W100C amplifier. The input power of these tones is adjusted to achieve a baseline distortion of −30.0 dBc at the amplifier output. Next, the first positive and negative switching harmonics, that is $k = \pm 1$, in addition to the fundamental tones at $k = 0$, are passed to the amplifier. The distortion drops to −44.7 dBc, a 14.7 dB reduction, which is in good agreement with the theoretical 14.5 dB reduction predicted by (12). Even for the highly non-ideal (i.e. sinusoidal) switching described by keeping only the $k = \pm 1$ switching harmonics, the IM3 is significantly reduced compared to that from two tones without any switching.

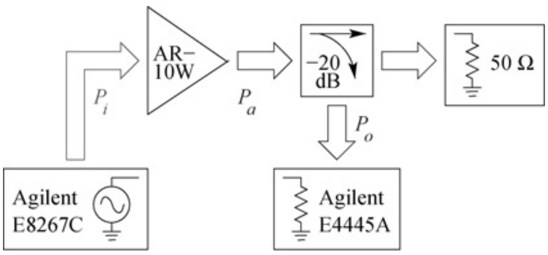


**Figure 3** *Test setup for $N = 2$ distortion reduction*

The Agilent E8267C generator has a digital-to-analogue sampling rate of 90 MS/s. The input power $P_i$ is adjusted so that $P_a$ is 16.2 dBm at each of the two fundamental tones

**Table 2** Distortion reduction by amplifying spectral replicas, $N = 2$

| Active $k$ | Measured distortion, dBc | Measured reduction, dB | Theoretical reduction, dB |
|---|---|---|---|
| 0 | −30.0 | N/A | N/A |
| −1, 0, 1 | −44.7 | 14.7 | 14.5 |
| −3, . . . , 3 | −48.5 | 18.5 | 20.1 |
| −5, . . . , 5 | −50.3 | 20.3 | 23.5 |
| −7, . . . , 7 | −51.2 | 21.2 | 26.0 |
| −9, . . . , 9 | −52.0 | 22.0 | 27.8 |
| −11, . . . , 11 | −52.7 | 22.7 | 29.5 |

Passing additional switching harmonics further reduces the IM3 seen at the amplifier output; however, the measured reduction will deviate from the theoretical reduction for higher values of $k$ because the frequency response of an amplifier is not flat over a wide bandwidth. When the frequency response is unbalanced, $a_0, \ldots, a_p$ are not necessarily symmetric about $k = 0$, that is the scaling of each term in the cancellation summation in (20) deviates from $1/k^2$ and the distortion does not approach zero as indicated by (14).

The gain of the AR10W100C varies by several decibels over the frequency span from $k = -11$ (corresponding to 399 MHz) to $k = +11$ (corresponding to 531 MHz). Thus, when all the harmonics between $k = -11$ and +11 are amplified, the distortion reduction deviates from the theoretical value by 6.8 dB. Still, the experiment indicates that a reduction in amplifier distortion of greater than 20 dB is possible by including spectral replicas of the fundamental tones at the switching harmonics $k = \pm 1$, $\pm 3$ and $\pm 5$ in the amplification. A reduction of greater than 14 dB is possible by including only one switching harmonic ($k = \pm 1$).

Additional measurements performed with a commercial radio-frequency integrated circuit (RFIC) power amplifier demonstrate the improvement in effective linearity of a practical amplifier when using LITMUS [8]. The RF micro devices (RFMD) RF2320 GaAs (gallium arsenide) metal semiconductor field effect transistor (MESFET) amplifier, with a specified OIP3 of 35 dBm and a P1dB output compression point of 23.5 dBm, replaces the AR10W100C amplifier in the next measurement reported. The LITMUS system is then characterised at several power levels with a time-multiplexed two-tone test signal for $k = 0$ (non-multiplexed) up through $k = \pm 13$. The tone spacing is again 100 kHz, centred at 465 MHz. The measurement results are summarised in Fig. 4. When compared to the application of two continuous tones, the results show a reduction in IM3 ranging from 16–28 dB for $k = \pm 1$ to $\pm 13$ at $P_o = 13$ dBm, and the improvement degrades by 4.7 dB as the output power increases to 17 dBm. The results demonstrate that LITMUS provides an improvement in linearity near compression and at backoff from compression.

## 4.2 Random phases, $N = 4$

Uniform-amplitude random-phase multisines have previously proven useful for analysing intermodulation products generated by non-linear transfer functions and practical signal spectra [9]. Such multisines were chosen to represent CDMA communication signals under non-linear amplification, for example [10]. In this section, a four-tone multisine whose phases are chosen randomly is used to



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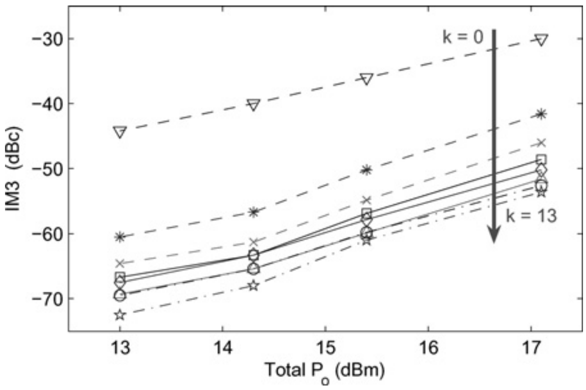


**Figure 4** *IM3 measurement data for the RF2320 amplifier*

Time-multiplexed two-tone test signal with $k = 0$ through $k = \pm 13$. A monotonic reduction in distortion is observed as the amplifier passes additional switching harmonics, for four different output power levels

mimic the behaviour of a communication signal carrying information by amplitude and phase modulation. This signal is multiplexed before amplification to demonstrate the advantage of grafting LITMUS onto a pre-existing communication scheme.

A measurement system which demonstrates distortion cancellation for four tones ($N = 4$) and random initial phases ($\phi_1, \ldots, \phi_4$) is given in Fig. 5. The baseband signal generator is the Agilent N6030A. Here, the Agilent E8267C acts as the RF modulator, which upconverts the signal from the N6030A to a carrier of $f_0 = 850$ MHz. The four tones are spaced 391 kHz apart and are time multiplexed with a switching frequency of $f_s = 39$ MHz. Only the first side lobe of the switching harmonics ($k = +4, \ldots, 4$) is generated by the E8267C waveform generator and input to the amplifier. The amplifier is the Ophir model 5162, which has a small-signal gain of $a_1 = 46$ dB. The spectrum analyser is again the Agilent E4445A.

For $N = 4$ (four tones) and a single lobe of switching harmonics ($k = -4, \ldots, 4$) applied to the amplifier, the input signal given by (6) becomes

$$v_i(t) = \sum_{k=-4}^{4} \frac{B_k}{\pi} \sum_{n=1}^{4} A_n \cos\left[(\omega_n + k\omega_s)\left(t - \frac{n-1}{4} T_s\right) + \phi_n\right] \tag{15}$$

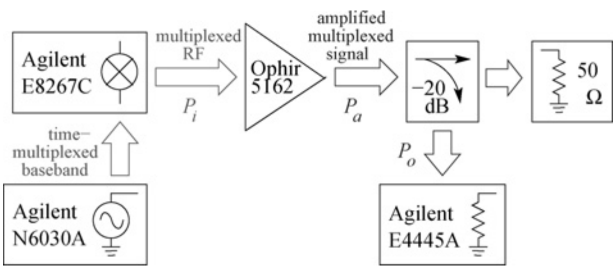


**Figure 5** *Test setup for random-phase distortion reduction*

For this experiment, the phases of the four tones $\phi_1, \ldots, \phi_4$ are randomised by the software controlling the Agilent N6030A. Each set of four random phases is independent of the others, and each of the four phases within each set is independent of the others. The phases are chosen from a uniform distribution between $-\pi$ and $+\pi$. Ten sets of four random phases are averaged from individual measurements to give the results shown in Table 3. A sample of the random-phase data for $P_o = 7$ dBm is given in Fig. 6. On average, using a single lobe of switching harmonics to multiplex the signal, LITMUS reduces the distortion for a randomly-phased four-tone signal by between 8.8 and 14.5 dB. These results are in good agreement with the 13.3 dB reduction for $N = 4$ and $k = -4, \ldots, 4$ predicted (see the Appendix). This demonstrates the applicability of LITMUS to reduce IMD generated by practical communication signals.

## 4.3 Wideband measurements, $N > 4$

Distortion reduction is also possible for wider signal bandwidths and signals with higher peak to average power ratios, representing signals of high-order modulation. To demonstrate cancellation for wider bandwidths containing greater numbers of tones, a wideband multisine measurement system was constructed. A block diagram of this system is given in Fig. 7. The input to the amplifier comes from the Polyphase Microwave QM3337A quadrature modulator, into which is fed the baseband time-multiplexed waveform from the Agilent N6030A wideband generator and a local oscillator signal from the Agilent E8267C. The tones are centred at $f_0 = 3562$ MHz, spaced $3.125/N$ MHz apart and time-multiplexed with a switching frequency of $f_s = 156.25$ MHz. The MiniCircuits ZHL-1042J is a high-linearity wideband amplifier that boosts the signal to a level sufficient to cause saturation in the Ophir 5162 amplifier, the device under test (DUT) in this setup. The Agilent E4445A again records the power spectra. The results of the wideband test are given in Table 4, and a sample of the wideband data for $N = 20$ is given in Fig. 8.

**Table 3** Measured random-phase distortion reduction, $N = 4$

| Output power $P_o$, dBm | Measured distortion, $k = 0$, dBc | Measured distortion, $k = -4, \ldots, 4$, dBc | Measured distortion reduction, dB |
|---|---|---|---|
| 9 | −46.5 | −61.0 | 14.5 |
| 8 | −49.2 | −61.2 | 12.0 |
| 7 | −52.4 | −62.6 | 10.2 |
| 6 | −56.4 | −65.2 | 8.8 |


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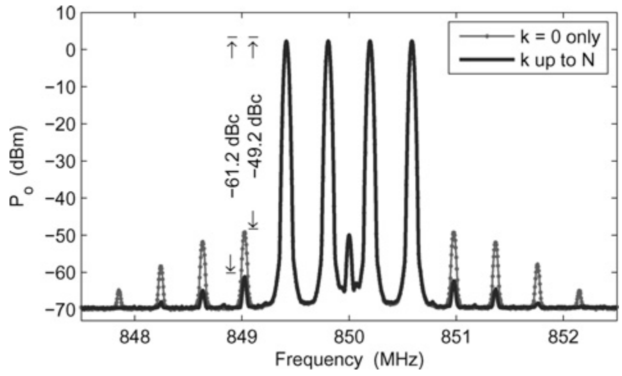


**Figure 6** *Sample random-phase spectra; data taken using the system of Fig. 5, N = 4 and $P_o = 7$ dBm total*

The distortion spectra produced by non-multiplexed ($k = 0$) and multiplexed ($k = -N, \ldots, N$) inputs to the amplifier are contrasted. An average distortion reduction of 12.0 dB is measured when time-multiplexing four tones with random phases, using the first side-lobe of switching harmonics. The signal at 850 MHz is an artifact of the arbitrary waveform generator

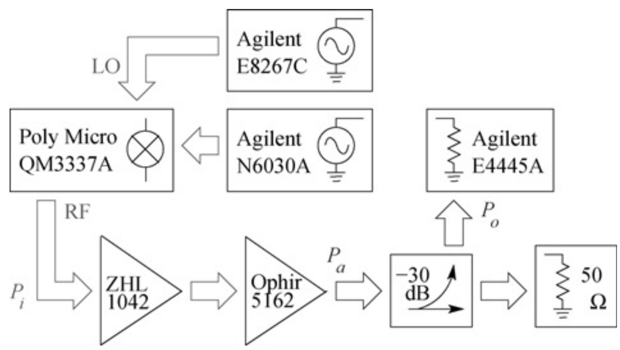


**Figure 7** *Test setup for wideband distortion reduction*

The Agilent N6030A generator has a digital-to-analogue sampling rate of 1.25 GS/s. The gain of the MiniCircuits ZHL-1042J amplifier is 27.6 dB at 3.562 GHz

The Agilent N6030A implements software filtering to control the number of switching harmonics applied to the amplifier. Initially, for each value of $N$, only the fundamental tones are applied to the Ophir 5162. The power of the N6030A quadrature channels is adjusted to achieve a baseline distortion of approximately −30.0 dBc at the amplifier output, and the adjacent channel power ratios (ACPRs) for the lower and upper sides of the 3.125-MHz-wide signal are recorded. Then a full set of switching harmonics up to $k = \pm N$ is added to the fundamental tones at the DUT input and the reduction in ACPR values from the $k = 0$ case is tabulated.

The data show that transmitting the full first lobe of the sinc-like switching waveform through the amplifier results in an ACPR reduction of between 13.9 and 16.1 dB from the non-multiplexed case for multisines up to $N = 20$. The reduction varies by about 2 dB because the gain responses of the MiniCircuits and Ophir amplifiers used here are not entirely flat over the 312.5 MHz bandwidth of the main multiplexed lobe. This non-ideal behaviour produces deviation from the ideal cancellation predicted. Still, the experiment demonstrates that LITMUS can provide distortion reduction of at least 13.9 dB in ACPR for an amplified signal containing $N = 20$ simultaneous frequencies within 3.125 MHz, thus demonstrating that LITMUS is applicable within a practical communication bandwidth.

**Table 4** Measured distortion reduction, wideband, $N > 4$, $k = -N, \ldots, N$

| Number of tones, $N$ | PAR, dB | ΔACPR, low side dB | ΔACPR, high side, dB |
|---|---|---|---|
| 6 | 7.8 | −14.4 | −13.9 |
| 8 | 9.0 | −16.1 | −15.9 |
| 10 | 10.0 | −14.8 | −14.1 |
| 12 | 10.8 | −14.5 | −14.6 |
| 14 | 11.5 | −16.0 | −15.6 |
| 16 | 12.0 | −15.3 | −14.8 |
| 18 | 12.6 | −14.7 | −14.7 |
| 20 | 13.0 | −15.3 | −15.2 |

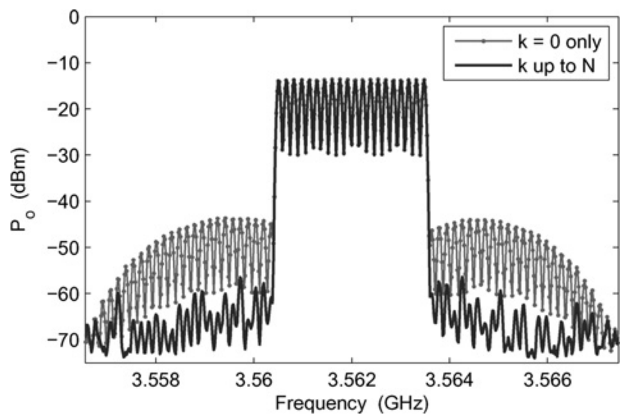


**Figure 8** *Sample traces; data taken using the system of Fig. 7, N = 20*

The distortion spectra produced by non-multiplexed ($k = 0$) and multiplexed ($k = -N, \ldots, N$) inputs to the amplifier are contrasted. A 15 dB reduction in distortion is observed experimentally when time-multiplexing 20 tones with the first side-lobe of switching harmonics

## 5 Linear signal recovery

If the amplified multiplexed signal is applied to a bandpass filter whose bandwidth is narrow enough to pass only the fundamental tones, the sampling aliases are eliminated and the switched-tone amplified output is converted back to its non-multiplexed form. This reconverted RF signal contains less distortion than the directly amplified multitone signal




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of equal tone power. This section presents theory and measurements which confirm these statements.

## 5.1 Theory

Let the transfer function of a bandpass filter that follows the amplifier be given by

$$H(\omega) = \frac{V_f(\omega)}{V_a(\omega)} = |H(\omega)| \exp\{j\theta(\omega)\} \tag{16}$$

The bandwidth of the filter is $B$ and it is centred on $k = 0$. If the bandwidth of the filter is larger than the non-multiplexed signal spectrum but smaller than the switching frequency ($\omega_N - \omega_1 < B < \omega_s$), the filter removes all spectral components above and below the fundamental tones, leaving only $k = 0$ at its output. If the amplifier small-signal gain is assumed to be a constant $a_1$ over the fundamental tones, then the spectrum of (4) after amplification and filtering (neglecting amplifier non-linearity, for the moment) may be written as

$$V_f(\omega) = \frac{a_1}{N} \sum_{n=1}^{N} \left\{ \begin{array}{c} A_n e^{j\phi_n} H(\omega_n) e^{-j\omega T_s[n-1]/N} \\ \times \pi[\delta(\omega - \omega_n) + \delta(\omega + \omega_n)] \end{array} \right\} \tag{17}$$

whose time-domain equivalent is

$$v_f(t) = \frac{a_1}{N} \sum_{n=1}^{N} A_n |H(\omega_n)| \times \cos\left(\omega_n t + \phi_n + \theta(\omega_n) - \omega_n \frac{T_s}{N}[n-1]\right) \tag{18}$$

The output is thus a superposition of sinusoids at the fundamental tones without time multiplexing. The amplitudes of the tones have been reduced by the switch duty cycle and the filter response, and increased by the amplifier gain. The phases of the tones depend upon the filter phase response and the time delay for each tone to be applied to the amplifier in the switching sequence.

If the amplifier passes all of the switching harmonics of $V_i(\omega)$, then the output contains no distortion because the switching of the tones is instantaneous, so that only one tone is present in the amplifier at any one time and no non-linear mixing occurs. In practice, however, every amplifier has a finite bandwidth, and every filter is weakly non-linear; these properties produce distortion in all outputs. Nonetheless, a switched-and-filtered signal will contain much less distortion than an unswitched signal that is amplified directly, for the same output tone power.

## 5.2 Measurement

A block diagram of the measurement system that demonstrates both the multiplexed to non-multiplexed conversion and the distortion reduction provided by this conversion is given in Fig. 9. The Agilent N6030A and E8267C units together form the signal generator. The signal consists of $N = 4$ tones, spaced 391 kHz apart, centred on $f_0 = 835$ MHz and time multiplexed with a switching frequency of $f_s = 39$ MHz. The filter is a basestation duplexer with a transmit band centred on 836 MHz, a nominal bandwidth of 25 MHz and an insertion loss of 1.2 dB. The Tektronix TDS684B oscilloscope records the time-domain waveforms, whereas the Agilent E4445A spectrum analyser records the frequency-domain data.

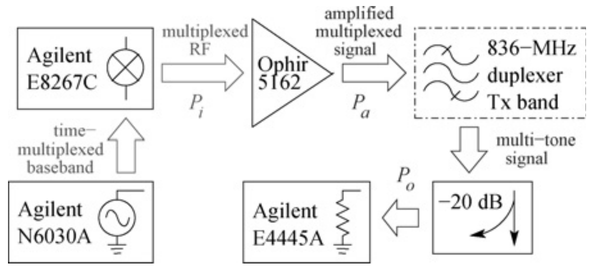


**Figure 9** *Test setup for demonstrating linear signal recovery from an amplified time-multiplexed signal*

The through-port of the 20-dB coupler is terminated in 50 Ω (not shown)

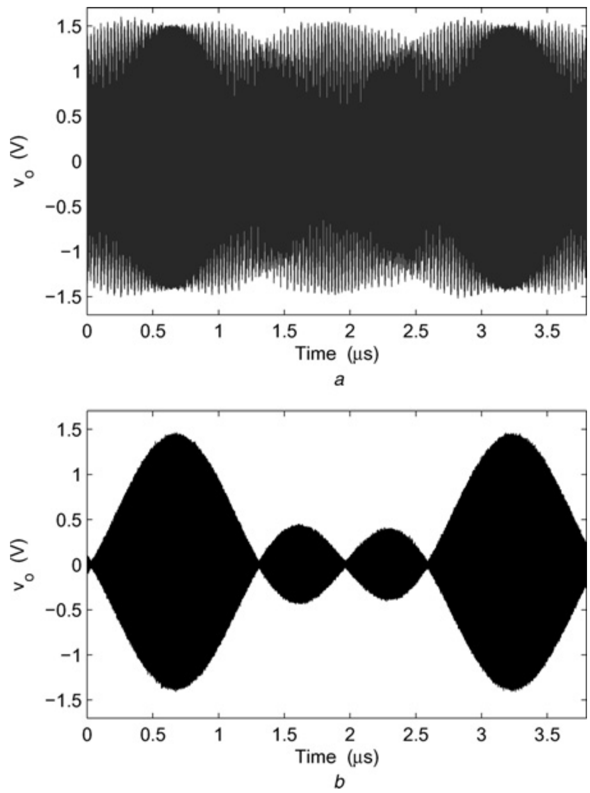


**Figure 10** *Measured time-domain comparison of switched-tone to multitone conversion*

*a* Amplified signal not filtered
*b* Amplified signal filtered using a low-loss duplexer
Four sinusoids are time-multiplexed to give the nearly constant-envelope signal in *a*. The filter de-multiplexes the sinusoids into the four-tone interference pattern in *b*

The switched tone to multisine conversion is demonstrated experimentally in Figs. 10 and 11. The switched-tone input to the amplifier $v_a$ is shown in Fig. 10*a* and its spectrum is displayed in Fig. 11*a*. This time-domain waveform has a nearly constant RF envelope and its spectrum is wideband. The residual modulation is due to the truncated switching spectrum; that is only switching harmonics with $k = -4, \ldots, 4$ are retained, so the switching is not ideal. The multitone output from the amplifier $v_o$ is shown in Fig. 10*b* and its spectrum is displayed in Fig. 11*b*. This time-domain waveform resembles a steady-state four-tone interference pattern and its spectrum is narrowband. The result given by (18) is thus confirmed experimentally.

The data in Fig. 12 illustrate the distortion reduction of the switched-to-multisine conversion. The spectrum of the filter output is plotted in the vicinity of the fundamental tones. The two traces are for (i) $k = 0$, which is the multisine without time-multiplexing and (ii) the main sinc lobe, which is the multisine produced by time-multiplexing and filtering. The filter is retained for the non-multiplexed case so that a comparison of distortion levels can be made for equal fundamental tone powers.

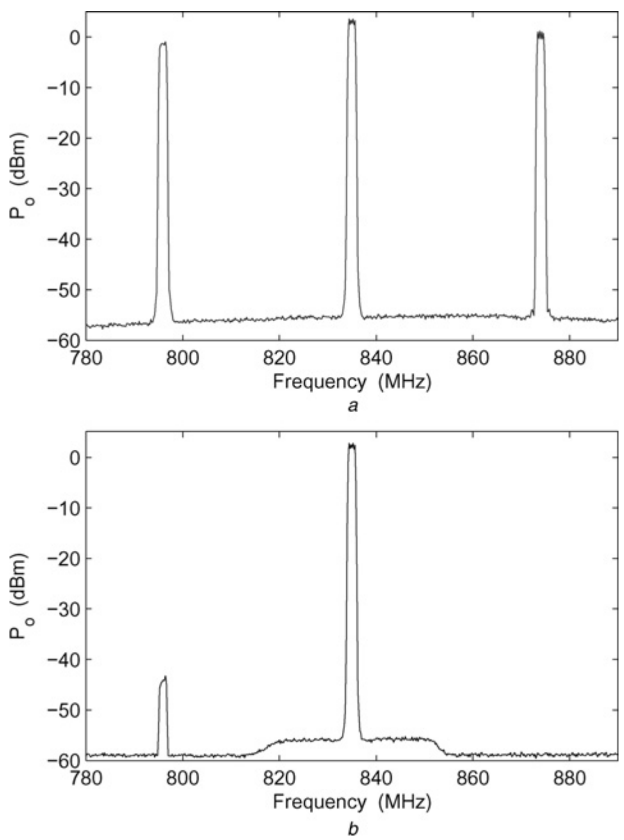


**Figure 11** *Measured frequency-domain comparison of switched-tone to multitone conversion*

*a* Amplified signal not filtered, $k = -1, 0, +1$ not attenuated
*b* Amplified signal filtered, $k = -1, 0, +1$ attenuated by the low-loss duplexer
There are four tones embedded in each visible peak

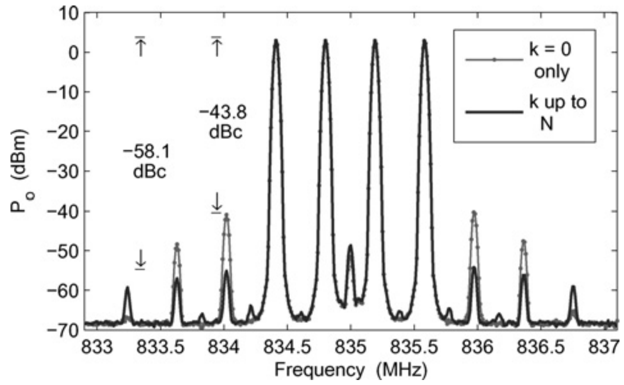


**Figure 12** *Sample traces; data taken using the system of Fig. 9 with N = 4*

The distortion spectra of the non-multiplexed ($k = 0$) and single-lobe multiplexed ($k = -N, \ldots, N$) outputs from the filter are contrasted. A 14.3-dB reduction in distortion is evident following the linear recovery of Figs. 10 and 11

The filtered time-multiplexed multisine contains 14.3 dB less distortion than its non-multiplexed counterpart. Since the RF envelope variation at the input to the amplifier is considerably reduced in the time-multiplexed case, the intermodulation produced by variations in gain over the period of the switched-tone signal is significantly reduced. The linearisation of an amplified signal by time-multiplexing it before the amplifier and filtering it after the amplifier is confirmed.

## 6 Conclusions

A novel waveform processing technique for reducing IMD of an amplified RF signal has been presented. Since a narrow-band digitally modulated signal can be represented by amplitude and phase components and since an amplitude-modulated signal can be represented as a sum of simultaneous sinusoids, a narrow-band digitally modulated signal containing amplitude modulation can be reconstructed from a sequence of short sinusoidal pulses. For the LITMUS technique, the duration of each pulse is proportional to its frequency's component amplitude and the phase of each pulse is proportional to its component phase. Time-separated components are amplified and then recombined to produce an amplified replica of a composite signal.

LITMUS demonstrates that time multiplexing the spectrum of an input signal at a rate greater than its bandwidth reduces its peak-to-amplitude ratio at the expense of momentarily widening the overall signal bandwidth. The reduced amplitude variation of the time-multiplexed signal spectrum generates significantly less distortion than the original signal. A bandpass filter is necessary to remove the wideband multiplexed aliases and recover the desired amplified signal.



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Theory predicts greater distortion cancellation when a greater number of switching harmonics are transmitted by the amplifier, assuming a flat gain characteristic. Measurements confirm that this distortion reduction is possible. A reduction of 14.5–22.7 dB is recorded for multiplexing two tones using $k = -1, \ldots, 1$ up to $k = -11, \ldots, 11$, and a reduction of 15.2 dB is recorded for multiplexing 20 tones using $k = -N, \ldots, N$. Measurements also confirm that the amplified signal may be recovered by using a bandpass filter to remove the time-multiplexing aliases. A distortion reduction of 13.7 dB is recorded for multiplexing and filtering four tones compared to amplifying the original signal without the filter.

It is easy to see in using LITMUS with an OFDM signal, the RF pulse train could consist of a first pulse for the first time-continuous OFDM modulated subcarrier, followed by a second pulse for the second subcarrier and so on. Design of the appropriate waveforms to be used with LITMUS to realise particular digital-modulation schemes remains to be developed.

# 7 Acknowledgments

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# 9 Appendix

## *9.1 Two-tone time-multiplexed distortion*

Upon writing the amplitude of the IMD for when the amplifier passes the switching harmonics $k = -7, \ldots, 7$, a pattern emerges

$$V_{\rm IM3} = \begin{bmatrix} \dfrac{3}{4} - \dfrac{6}{\pi^2} - \left(\dfrac{6}{\pi^2}\right)\left(\dfrac{1}{9}\right) \\ -\left(\dfrac{6}{\pi^2}\right)\left(\dfrac{1}{25}\right) - \left(\dfrac{6}{\pi^2}\right)\left(\dfrac{1}{49}\right) \end{bmatrix} a_3 A^3 \quad (19)$$

The IMD as the amplifier passes the switching harmonics $-\eta, \ldots, \eta$ is thus

$$V_{\rm IM3} = \left[\frac{3}{4} - \frac{6}{\pi^2} \sum_{\substack{k=1 \\ k\ \rm odd}}^{\eta} \frac{1}{k^2}\right] a_3 A^3 \quad (20)$$

The sum may be rewritten using the relationship

$$\sum_{\substack{k=1 \\ k\ \rm odd}}^{\eta} \frac{1}{k^2} = \frac{3}{4} \sum_{k=1}^{\eta} \frac{1}{k^2} \quad (21)$$

such that (20) becomes

$$V_{\rm IM3} = \frac{3}{4}\left[1 - \frac{6}{\pi^2} \sum_{k=1}^{\eta} \frac{1}{k^2}\right] a_3 A^3 \quad (22)$$

Evaluating the sum as $\eta \to \infty$ gives

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \zeta(2) = \frac{\pi^2}{6} \quad (23)$$

where $\zeta(2)$ is the Riemann zeta function evaluated at power 2.


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Substituting this result into (22) gives

$$\lim_{\eta\to\infty}(V_{\mathrm{IM3}}) = \frac{3}{4}\left[1-\frac{6}{\pi^2}\left(\frac{\pi^2}{6}\right)\right]a_3A^3 = 0 \qquad (24)$$

## 9.2 Four-tone time-multiplexed distortion

For $N = 4$ and $k = -4, \ldots, 4$, Table 1 becomes Table 5. The amplitudes and phases correspond to spectral content at the frequency $2\omega_4 - \omega_1$. The '$\Delta\phi$' row is added because some IM3 components have phases between 0 and $\pm\pi$. Here, the '$V_{\mathrm{IM3}}$' row is the sum of all $\beta-\delta$ combinations for a particular $\alpha$ which have the same phase $\Delta\phi$.

As with $N = 2$, there are some $\alpha-\beta-\delta$ combinations which add to the non-multiplexed distortion (i.e. the $58/9\pi^2$ in the $\alpha = 0$ column). Some combinations subtract from the distortion with phases of $\pi$ (i.e. the $\alpha = \pm 2$ column). The rest of the combinations are offset in phase from the $\alpha = \beta = \delta = 0$ distortion by multiples of $\pi/2$, but they cancel each other in pairs. The $\alpha = -1$ terms are offset by $+\pi/2$, whereas the $\alpha = +1$ terms are offset by $-\pi/2$. Because each set has the same series of amplitudes, they cancel. The $\alpha = -3$ and $+3$ components add to zero the same way. The $k = \pm 4$ combinations do not add to or subtract from the distortion because at least one sinc function goes to zero if any of the three indices equals $N$.

**Table 5** Time-multiplexed IM3, $N = 4$ and $k = -4, \ldots, 4$

| $\alpha$ | 0 | $\pm 1$ | $\pm 2$ | $\pm 3$ | $\pm 4$ |
|---|---|---|---|---|---|
| $\beta+\delta$ | 0 | $\pm 1$ | $\pm 2$ | $\pm 3$ | $\pm 4$ |
| $V_{\mathrm{IM3}}$ | $\frac{3}{4}+\frac{58}{9\pi^2}$ | $\frac{16}{\pi^2}+\frac{104}{3\pi^3}$ | $\frac{4}{\pi^2}+\frac{20}{3\pi^3}$ | $\frac{4}{9\pi^2}+\frac{8}{3\pi^3}$ | 0 |
| $\Delta\phi$ | 0 | $\frac{\pm\pi}{2}$ | $\pi$ | $\frac{\pm\pi}{2}$ | 0 |

Using $k = -4, \ldots, 4$, the total (normalised) distortion is

$$V_{\mathrm{IM3}} = \frac{3}{4}+\frac{58}{9\pi^2}-\left[\frac{40}{3\pi^3}+\frac{8}{\pi^2}\right] \qquad (25)$$

$$= \frac{3}{4}\left(1-\frac{56}{27\pi^2}-\frac{160}{9\pi^3}\right) \qquad (26)$$

which is a 13.3-dB reduction in IMD from $k = 0$.

As with $N = 2$, for higher $k$ values, the number of combinations that are $\pm\pi$ out-of-phase with the distortion outpaces the number of $\alpha-\beta-\delta$ combinations which add to the non-multiplexed distortion and the theoretical limit as the non-linearity passes an infinite number of switching harmonics is $V_{\mathrm{IM3}} = 0$.

For higher values of $N$ and one full sinc-lobe of switching harmonics $k = -N, \ldots, N$, the span of phases $-\pi, \ldots, \pi$ becomes increasingly subdivided across these $\alpha-\beta-\delta$ sets. With $N$ tones, the terms are offset from each other by multiples of $2\pi/N$. For even $N$, there will exist $\alpha-\beta-\delta$ sets which cancel the original distortion at rotations of exactly $-\pi$ and $+\pi$, whereas the other sets cancel themselves in pairs. For odd $N$, the distortion will not cancel with phase shifts of exactly $\pm\pi$, but overall it will still be reduced.



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