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Filter characterisation using one-port pulsed radio-frequency measurements

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Abstract: A method for extracting the loaded quality factor of the first resonator of a bandpass filter using one-port measurements is introduced. Properties of the quality factor are reviewed, and a time-domain approximation is used to estimate the loaded Q value from the decay rate of the filter's natural response. Narrowband 900 MHz Chebyshev filters are characterised.

1 Introduction

As radio-frequency (RF) device manufacturers reduce the size, cost and time-to-market of wireless communications products, higher levels of integration and more sophisticated automated testing procedures are being employed. Bandpass filters, as with many other front-end components that once were produced discretely, are now manufactured as parts of integrated assemblies. Technologies enabling three-dimensional integration, such as low-temperature cofired ceramics (LTCC), have reduced the real-estate occupied by frequency-selective components [1].

A drawback of vertical integration, however, is that there are few access points for testing individual subsystems. Multilayer filters based on LTCC [2], for example, offer a compaction of the filter structure at the expense of access to its individual resonators. As integrated front-ends have fewer testing sites available, identification of those components or subsystems which do not meet specifications becomes increasingly difficult.

A filter's frequency selectivity is often skewed by variations in manufacturing because of materials and packaging. Thus, manual or automated tuning is often necessary. To enable tuning and to avoid disassembling RF front-ends in order to test component functionality, non-destructive methods of probing integrated filters are sought.

This paper addresses the extraction of filter characteristics using single-port pulsed RF measurements. Such methods would allow the components of a filter to be tested without the need to disassemble an integrated design. Recent work employing swept stimuli has shown that it is possible to characterise the passband of a filter by examining intermodulation content reflected back through the filter by nonlinear components deeper into the front-end [3]. Present work employs single-tone pulses to show that it is possible to estimate the loaded quality factor of the first resonator of a bandpass filter resonator chain by examining its linear reflection alone. A filter has minimum in-band reflections only in steady state, and when an in-band RF pulse is applied, there are reflections until internal resonators are 'charged.' The technique reported here exploits the characteristic transient of the envelope of the reflected RF signal.

2 Background

Several one-port techniques for extracting loaded and unloaded quality factors of individual resonators have been developed [4–13]. Each method assumes that the circuit under test can be modelled as a single RLC resonator at frequencies near its resonant modes.

Ginzton [4] introduced two graphical methods of measuring quality factor Q, using a slotted line: the first technique finds the half-power points on the voltage standing wave ratio (VSWR) against the frequency curve and uses a fractional-bandwidth relationship to determine Q; the second technique plots the reflection coefficient in the complex plane and uses a series of frequency markers along the circle traced out by the resonance to determine Q. Sucher and Fox [5] reviewed Ginzton's methods and determined the resonator coupling and loss conditions under which his measurements are valid. Building on Ginzton's graphical methods to trace loci of Q factors on the Smith Chart®, Khanna and Garault [6] were able to determine the unloaded quality factor of a dielectric resonator coupled to a microstrip line in a bandstop configuration.

Shortly thereafter, Kajfez and Hwan [7] showed that the required measurements could be performed on a network analyser. They augmented the RLC circuit model to account for additional energy storage and loss introduced by the analyser's probes. By mapping the parameters of the augmented circuit to a series of coefficients that trace out a circle on the Smith Chart®, Kajfez [8] was also able to extract its unloaded resonant frequency and its coupling coefficient.

Sun and Chao [9] changed the circuit model by using Foster's equivalent forms. They devised the 'critical points' method, a method for extracting Q by identifying the critical points of the input reactance against the frequency curve. Chua and Mirshekar-Syahkal [10] extended this work to extract the coupling coefficient and the loaded resonant frequency from the same measurement.

Time-frequency methods for extracting quality factor appeared in the 1990s. Exploiting advances in computing power, Wang *et al.* [11] created discrete models of resonant structures using Yee's finite difference time-domain method. They were able to estimate Q by translating time-stepped data to the frequency domain using the Fast Fourier Transform and the previously established relationships between Q and the fractional bandwidth. To reduce computation time, Pereda *et al.* [12] substituted damped complex exponentials for the traditional lossless exponentials in the basis functions of the transform.

This paper introduces a purely time-domain method for extracting the loaded quality factor of the first resonator in a chain of resonators by measuring the time constant of the resonator's energy decay. The present technique is based on the decrement method introduced by Ginzton [4], which was originally used to estimate the Q of a two-port resonator from its time-domain transmission response. The waveforms from which Q is extracted are similar to those reported by Dunsmore [13], although the time-domain data presented here are captured directly from a sampling oscilloscope rather than transformed from frequency-domain network analyser sweeps. The present technique is limited by available signal generation hardware to narrowband designs; however, it is theoretically applicable to filters of any bandwidth, and it requires only a single measurement port and a single oscilloscope trace recorded at or near the device's resonance.

3 Quality factor

To understand the utility of the one-port transient resonator characterisation, it is necessary to review the properties of the quality factor, Q. Also, to avoid confusion, a distinction must be made between the 'unloaded' or 'ideal' quality factor normally denoted by Q_U or Q_0, which evaluates a resonant structure without resistive loading, and the 'loaded' or 'effective' quality factor denoted by Q_L or Q_{eff}, which evaluates a resonant structure after it has been integrated into a larger circuit. Throughout the following discussion, Q will refer to the loaded quality factor assuming 50 Ω resistive terminations.

Q is a measure of the energy storage capability of a resonant structure. In a circuit, it relates the average energy stored in electric and magnetic fields to the energy dissipated by the system, scaled by the circuit's resonant frequency [14]

$$Q = \omega \frac{(\text{average energy stored})}{(\text{energy loss/second})} \quad (1)$$

Q can range from 0 – representing a circuit which stores no energy, up to ∞ – representing a resonant circuit which is lossless.

Q is most commonly calculated in the frequency domain as the inverse of the resonant circuit's half-power percentage bandwidth

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{1}{\gamma} \quad (2)$$

where $\Delta\omega$ is the difference between the upper and lower corner frequencies and ω_0 the centre frequency of the filter. For second-order circuits, the value given by (2) is exact but for higher-order circuits, the value given by (2) is an approximation. Using this fractional-bandwidth relationship, the quality factor of a bandpass system from scattering parameters can be determined in a straightforward manner.

If such frequency-domain information is not available, however, Q can be estimated in the time domain by approximating the circuit's natural response as a first-order exponential drop. Q may also be calculated as 2π times the number of RF cycles required for the system's stored energy to drop to $1/e$ of its initial value, or [15]

$$Q = 2\pi \frac{\tau}{T_0} = \omega_0 \tau \quad (3)$$

where τ is the time constant of the energy decay and T_0 the period of the RF excitation at the circuit's resonant frequency. This time-domain approximation proves useful when the quality factor of a single resonator is sought.

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4 Simulations

Simulations have shown that if the input port of a Chebyshev filter is excited by a single-tone signal of constant amplitude, the loaded Q of the filter's first resonator can be approximated by measuring the rate of energy decay immediately after the stimulus tone is turned off.

The simulated circuit is that of Fig. 1 with voltage measured at Port 1. The input waveform is a single-tone sinusoid at the filter's centre frequency ω_0 with constant amplitude $2V_0 = 1.0$ V. The source is turned off at time $t = 0$. A plot of the simulated voltage envelope for three filters with different bandwidths is given in Fig. 2, and shows characteristic decay forms.

The time constant of the energy decay is found by approximating the beginning of the voltage decay by the exponential function

$$V_R(t) = V_0 e^{-t/2\tau} \quad (4)$$

where V_R is the magnitude of the voltage wave across the input resistance R, and V_0 the magnitude of the steady-state voltage across the input port immediately before the input stimulus is turned off. Sinusoidal variation at the filter's centre frequency is implied; only the envelope of the voltage wave is analysed. Justification for this first-order exponential approximation of the envelope response is provided in Section 5.

The energy decay is equal to the power dissipated in the source termination

$$P(t) = \frac{V_R^2(t)}{R} = \frac{V_0^2}{R} e^{-t/\tau} \quad (5)$$

Taking the natural logarithm of both sides and rearranging the terms gives

$$\tau = \frac{t}{\ln(V_0^2) - \ln(V_R^2(t))} \quad (6)$$

where t must be chosen close to the start of the decay. The Q factor can then be estimated from (3).

Table 1 lists this estimated Q for several simulations of varying filter order, bandwidth and passband ripple. Alongside the estimated Q is the loaded Q of the first resonator calculated using the capacitance and inductance

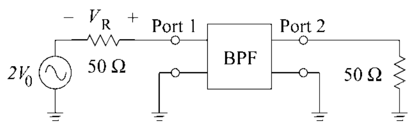

Figure 1 *Probe circuit for 1- and 2-port Q-value estimation*

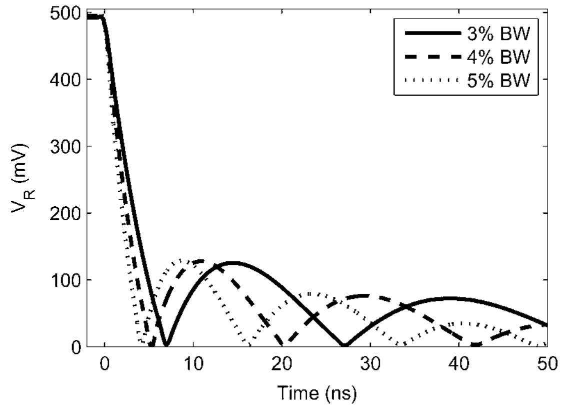

Figure 2 *Simulated envelope decay at filter input port: Chebyshev designs, 900 MHz centre frequency, 0.01 dB passband ripple, varied bandwidth, 900 MHz stimulus tone turned off at t = 0 (carrier not shown)*

of the lumped-element Chebyshev simulation model and a 50 Ω resistance. Although the estimated Q consistently under-approximates the resonator Q, the maximum deviation between the estimated and exact values is $<6\%$.

Table 1 Simulated Q-value estimation, 900 MHz Chebyshev design

Filter order	BW %	Passband ripple, dB	Loaded *Q* value	Estimated *Q* value
3	3	0.01	21.0	20.5
	3	0.10	34.4	33.2
	4	0.01	15.7	15.3
	4	0.10	25.8	25.0
	5	0.01	12.6	12.2
	5	0.10	20.6	20.0
5	3	0.01	25.5	24.9
	3	0.10	38.2	36.5
	4	0.01	19.1	18.7
	4	0.10	28.7	27.5
	5	0.01	15.3	14.9
	5	0.10	22.9	22.1
7	3	0.01	26.6	25.9
	3	0.10	39.4	37.0
	4	0.01	19.9	19.4
	4	0.10	29.5	28.8
	5	0.01	15.9	15.5
	5	0.10	23.6	22.6

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5 Mathematical modelling

5.1 Low-pass prototyping

To show why this one-port Q estimation is valid, low-pass prototyping is invoked. A seventh-order lumped-element Chebyshev filter and its low-pass equivalent are shown in Figs. 3*a* and 3*b*, respectively.

Typically, bandpass filter design begins with the filter's low-pass prototype, a network of series inductances and shunt capacitances, realising specifications of rolloff, passband ripple and 1 Ω resistive terminations. A frequency transformation is then used to centre the filter's passband at the operating frequency and to accommodate a change in resistance at the filter's input and output ports.

The bandpass transformation substitutes a series LC combination for each L_N and a parallel LC combination for each C_N. The relationship between the first resonator of the bandpass filter and the first capacitor of its low-pass prototype is as follows [16]

$$L_1 = R\frac{\gamma}{C_{N1}\omega_0} \quad C_1 = \frac{C_{N1}}{R\omega_0\gamma} \quad \omega_0 = \frac{1}{\sqrt{L_1 C_1}} \tag{7}$$

where R is the resistance of the input and output terminations, and ω_0 the centre frequency of the resonator as well as that of the overall filter.

It is also noted in [16] that a simple time-scaling relationship exists between (a) the envelope of the transient response of a bandpass filter, when excited by a step-modulated waveform with carrier frequency equal to the filter's centre frequency, and (b) the transient response of the filter's low-pass equivalent, when excited by a step change in the voltage. An illustration of this relationship is shown in Fig. 4.

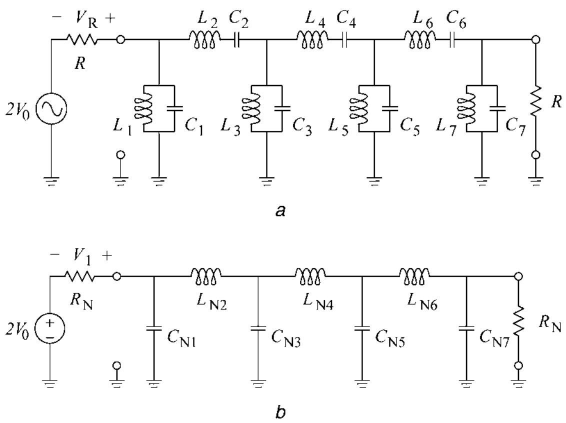

Figure 3 *Seventh-order Chebyshev filter*

a Lumped element model
b Low-pass equivalent circuit

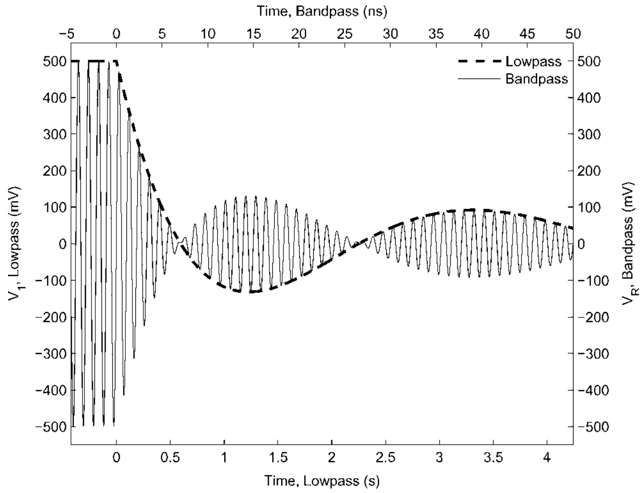

Figure 4 *Time-scaling relationship between bandpass filter and low-pass prototype transient responses: seventh-order 0.01 dB ripple Chebyshev designs, simulated*

Low-pass filter has a corner frequency of 1 rad/s and the bandpass filter has a centre frequency of 900 MHz and a bandwidth of 27 MHz
Source voltage $2V_0$ is deactivated at $t = 0$

The envelope of the voltage wave across the input resistor in the bandpass case is a time-scaled version of the same voltage wave in the low-pass case

$$t = \alpha\, t_N \quad \alpha = \frac{2}{\omega_0\gamma} \tag{8}$$

where t is the time scale of the bandpass response and t_N is the time scale of the low-pass equivalent response.

The following analysis shows that the Q factor derived from considering the filter's bandpass envelope is equal to the Q factor derived from steady-state considerations, with the approximation that the decay of the low-pass equivalent is a first-order exponential drop.

5.2 Time-frequency equivalence of Q

From (1), the quality factor of the first resonator from steady-state considerations alone is

$$Q_1 = \omega_0\frac{(1/2)C_1V_1^2}{(1/2)V_1^2/(R/2)} = \frac{R}{2}\frac{1}{\sqrt{L_1C_1}}C_1 = \frac{R}{2}\sqrt{\frac{C_1}{L_1}} \tag{9}$$

with two parallel resistive branches R attached to the RLC tank circuit.

To determine Q in the time domain, the differential circuit equation involving the first reactive component in the low-pass prototype is considered. The voltage V_1 measured

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across the input resistance R_N in Fig. 3*b* is found by solving

$$\frac{V_1(t_N)}{R_N} + C_{N1}\frac{dV_1(t_N)}{dt} + I_2(t_N) = 0 \qquad (10)$$

It should be noted that this expression is strictly valid for all odd-order filters. For even-order filters, a sum-of-voltages replaces (10) and the same result for Q is obtained. There is no loss of generality.

When the source voltage is turned off at $t_N = 0$, the circuit becomes that of Fig. 5*a* Since the current through inductor L_{N2} cannot change instantaneously, its value immediately before and after the input is turned off is a constant, and since current must flow through L_{N2} before reaching the circuit components nearer to the output, initially only capacitor C_{N1} reacts to the change. Thus, as a consequence of the cascaded structure, initially only the voltage across C_{N1} decays, whereas the other reactive elements retain their 'charged' voltage and current values. From the standpoint of C_{N1}, the circuit immediately after the switch is that of Fig. 5*b*, and (10) becomes

$$\frac{dV_1(0)}{dt_N} = -\frac{V_1(0)}{R_N C_{N1}} - \frac{I_2(0)}{C_{N1}} = -\frac{2}{R_N C_{N1}}V_1(0) \qquad (11)$$

whose solution, in the neighbourhood of $t_N = 0$, is

$$V_1(t_N) = V_0\, e^{-2t_N/R_N C_{N1}} \qquad (12)$$

It is important to note that the same exponential factor is obtained by analysing the transient behaviour at the beginning of the voltage step rather than at the end; and therefore this same technique can be used to extract Q from the start of the RF pulse.

The power dissipated in the 1 Ω resistor R_N is

$$P(t_N) = \frac{V_0^2}{R_N} e^{-4t_N/R_N C_{N1}} = V_0^2 e^{-t_N/\tau_N} \qquad (13)$$

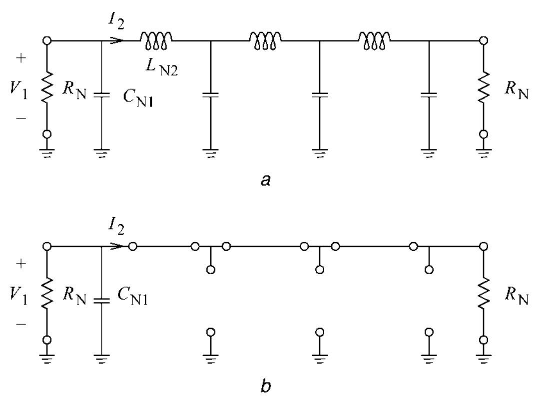

Figure 5 *Equivalent low-pass circuit after source is zeroed*
a For all time
b Immediately after zeroing

with a low-pass time constant

$$\tau_N = \frac{C_{N1}}{4} \qquad (14)$$

Time-scaling (14) by (8), the bandpass time constant is

$$\tau = \alpha\tau_N = \frac{C_{N1}}{2\omega_0\gamma} \qquad (15)$$

which, when substituted into (3) and expanded by (7), gives

$$Q = \frac{C_{N1}}{2\gamma} = \frac{1}{2\gamma}R\left(\frac{1}{\sqrt{L_1C_1}}\right)\gamma C_1 = \frac{R}{2}\sqrt{\frac{C_1}{L_1}} \qquad (16)$$

This value is equal to the steady-state quality factor given by (9). The time-domain envelope-decay calculation of Q gives the same result as the traditional steady-state frequency-domain calculation of Q.

6 Measurements

Measurements confirm that the loaded Q of the first resonator may be extracted using the estimation method described by probing the bandpass filter with an RF pulse generator and capturing the necessary time-domain data with a sampling oscilloscope.

It should be noted that this measurement technique is valid only if (a) the rate of decay of the stimulus pulse is much greater than the rate of decay of the resonator under test and (b) the oscilloscope used to record the reflected waveform is able to produce a faithful image of the resonator's decay. These conditions imply that the RF bandwidths of both the pulse generator and oscilloscope must be significantly greater than the bandwidth of the filter.

Using a vector signal generator with a closed-loop bandwidth of 70 MHz, two filters were analysed: one with a bandwidth of 36 MHz and the other with a bandwidth of 45 MHz. The nominal centre frequency of both the filters was 900 MHz. A block diagram of the measurement system is given in Fig. 6.

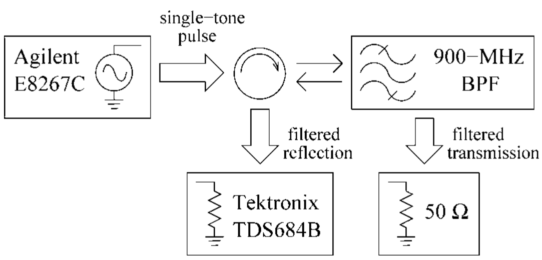

Figure 6 *Block diagram of filter decay measurement system: the single-tone pulses have an amplitude of 1 V for a period of 1 μs, followed by 0 V for 1 μs; the circulator is rated from 650 MHz to 1 GHz; the sampling rate of the oscilloscope is 5 GS/s*

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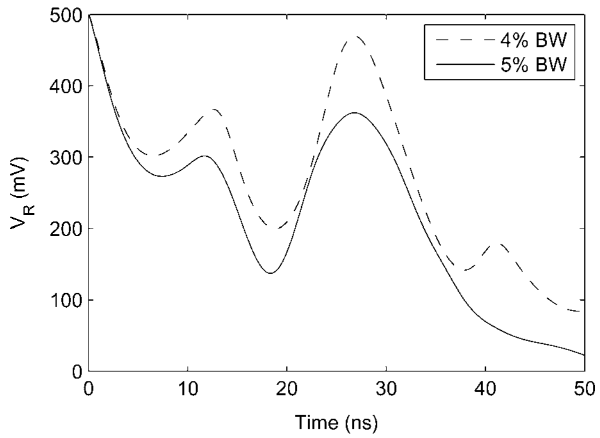

Figure 7 *Measured envelope decay at filter input port: Chebyshev designs, 900 MHz centre frequency, varied bandwidth, stimulus tone turned off at* $t = 0$

For Q calculation, $V_0 = 500$ mV and $t = 200$ ps

With the Agilent E8267C digital synthesiser as the waveform source, the input of each filter was pulsed with 1 μs bursts of a 1.0 V sinusoidal source at 900 MHz. A circulator was inserted between the source and the filter input port such that the reflected voltage wave from the filter was redirected to the oscilloscope. The Tektronix TDS684B digitising oscilloscope recorded the raw voltage waveforms. A Matlab® routine was then used to remove the 900 MHz carrier from the raw oscilloscope data, leaving only the characteristic envelope of each waveform. (Alternatively, a rectifier and low-pass filter could be used to capture the signal's envelope.) Fig. 7 shows two sample traces of these waveform envelopes. The traces are truncated so that only the envelope response immediately following the sinusoidal burst is seen. These traces are analogous to the simulated filter responses of Fig. 2.

The equation form at the start of the voltage decay is assumed to be that of (4). V_0 is equal to the value of the first voltage sample for each trace. Time $t = 0$ corresponds to the moment that this first sample is recorded. The energy decay time constant τ is determined from (6), where t is the time that the second sample is recorded. Q is then calculated using (3).

Table 2 compares the one-port Q-value estimation against the two-port fractional-bandwidth Q-value measurements of two Chebyshev filters with different bandwidths. The one-port estimations were found to be within 10% of their corresponding two-port measurements. These values are consistent with the simulation results for seventh-order filters with a passband ripple of 0.01 dB.

Table 2 Measured Q-value estimation, 900 MHz Chebyshev design

Filter order	BW, %	Passband ripple, dB	Two-port Q value	One-port Q value
7	4	0.01	23.3	24.6
	5	0.01	18.4	20.1

7 Discussion

Further insight is obtained by developing an understanding of how the construction of a bandpass filter shapes its transient responses.

Bandpass filters are made up of coupled resonators. Inside each filter, individual sections with similar resonant properties are chained together to form a larger structure whose frequency selectivity meets a particular design criterion. When an RF pulse is first applied to the filter input, the resonators charge to their steady-state voltages and currents. For frequencies outside the passband, the pulse returns from the input port unchanged, little energy is stored in the resonant structure and very little energy is transmitted through to the output. For frequencies inside the passband, most energy enters the filter, the maximum amount of energy is temporarily stored in the resonators and most energy is eventually transmitted through to the output port.

When the pulse is removed, the resonators release their stored energy to the terminal resistances, as well as to internal parasitics. The rate of energy decay is set by both the energy storage capacity of the filter (e.g. the number and size of the reactive elements) and the resistances of the paths by which the resonators discharge. Since the first resonator in the filter chain is physically nearest to the input port, the energy decay measured immediately after the source is shut off will most nearly follow the circuit time constant of the first resonator. As time progresses, energy from the internal resonators also passes out of the input port; their different Q factors and multiple rates of decay may help explain the ripple pattern seen after the initial decay.

8 Conclusions

A single-port probing method for estimating the loaded quality factor of a filter's outermost resonator has been introduced. Previous techniques were frequency-domain-based and were unable to isolate a single resonator. The most relevant methods for calculating Q still required measurements at multiple frequencies. The present method is able to characterise a single resonator, while requiring only a single reflective trace recorded at a single excitation frequency. This derives from the equivalence of the time- and frequency-domain definitions of Q. The method provides a fast low-power inexpensive quality assurance test.

A comparison of the natural responses of a bandpass filter and its low-pass prototype showed that a resonator's envelope response may be written in a simple exponential form, and its quality factor may be determined in the time domain. Measurements on cellular-band filters confirm this result.

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Where access to the individual components of a bandpass system becomes so limited as to allow only a single port for testing, this time-domain method is adequate to characterise the first resonator in a filter chain.

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