Estimation of co-channel nonlinear distortion and SNDR in wireless systems

K.M. Gharaibeh, K.G. Gard and M.B. Steer

Abstract: The effective signal-to-noise and distortion ratio (SNDR) at the output of a nonlinear amplifier is defined through the decomposition of the nonlinear output into correlated output and uncorrelated distortion. The analysis is based on the orthogonalisation of the nonlinear behavioual model that allows the accurate estimation of the effective in-band (or co-channel) distortion and hence determination of SNDR. Fundamental issues regarding the evaluation of the effective in-band distortion and its effect on digitally modulated signals are discussed. Simulations of in-band distortion and SNDR of WCDMA signals are verified experimentally using feed-forward cancelation.

1 Introduction

Nonlinear distortion from wireless transmitters results in two system impairments: the first is co-channel interference and the second is adjacent channel interference (ACI). ACI distortion has received much attention in analysis and measurement of wireless components mainly because of the ease in which it is measured from directly comparing out-of-band distortion to the desired signal. Co-channel distortion is more difficult to quantify because it is not directly observable from the spectral measurement of the output signal. Alternatively, co-channel distortion is quantified from measurements waveform signal quality metrics such as signal-to-noise and distortion ratio (SNDR) error vector magnitude (EVM) and the correlation coefficient (ρ). These signal metrics are measured using a digital receiver to demodulate the output signal and then perform time-domain calculations using the input signal. Demodulation and determination of signal metrics from circuit simulation of complex integrated circuit designs is not practical. Therefore, new analysis techniques are needed to determine efficiently signal metrics from circuit simulations and laboratory measurements.

The key to understand the relationship between co-channel distortion and SNR is to recognise that interference is defined by the total in-band signal power that is uncorrelated with the desired signal. Thus, the SNR at the output of a nonlinear circuit is determined by the ratio of the desired signal to the uncorrelated in-band distortion and noise.

The identification of the uncorrelated distortion components is usually done by assuming that the input signal has a Gaussian statistical properties. Then, using Bussgang theorem [1], the response of a general nonlinearity to a Gaussian process consists of an amplified replica of the input signal and an uncorrelated distortion component [2–4]. The Gaussian assumption leads to a simplified analysis of distortion in CDMA systems; however, it is not always valid to model CDMA wireless communication signals which exhibit statistical properties that depend on number of active users, user power profile, modulation and coding [5]. A theoretical analysis of the decomposition of the output spectrum into uncorrelated components without using the Gaussian assumption was studied in [5–12] where on the basis of the properties of the distribution function of the input signals, the output of a bandpass nonlinearity can be expressed as a sum of uncorrelated components.

In [13], we presented an orthogonalisation procedure of the nonlinear behavioural model that enables the effective in-band distortion in IS-95 system to be estimated. In this paper, we present a generalised approach for the accurate estimation of in-band distortion introduced by nonlinear amplification in WCDMA systems. We present an orthogonalisation procedure on the basis of Gram–Schmidt orthogonalisation for the nonlinear model to determine the effective in-band distortion. Gram–Schmidt orthogonalisation is more general than orthogonal polynomials to model nonlinearity [14, 15], because it does not impose a certain probability distribution on the input waveform. Measurement verification of the predicted uncorrelated inband distortion in WCDMA systems is achieved using feed-forward cancelation of the desired signal at the output of the nonlinear amplifier. The estimated effective in-band distortion obtained from the orthogonalised model is used to predict the effective SNDR (which is directly related to BER for a particular modulation scheme). Predicted in-band distortion is in very good agreement with measured in-band distortion and SNDR.

2 Orthogonal behavioural model development

Many nonlinear models are available for behavioural modelling of nonlinear circuits; however, the developed model presented here is intended to provide insight into how nonlinear in-band distortion contributes to signal-to-noise and distortion degradation. To this end, the behavioural model is developed by orthogonalising the nonlinear response in respect to the input signal and each branch of the model such that each component at the output of the model is orthogonal to each other. In the proposed model, a complex power series behavioural model of the nonlinear

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doi:10.1049/iet-map:20070034

Paper first received 7th October 2006 and in revised form 31st May 2007

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circuit is utilised to represent a quasi-static, or memoryless, nonlinearity, although the procedure presented is generally applicable to more complicated models.

A memoryless nonlinearity can be characterised as a power series model of the form

$$y(t) = \sum_{n=1}^{N} y_n(t) = \sum_{n=1}^{N} a_n w^n(t) \tag{1}$$

This representation however does not guarantee the representation of the output as pure linear and pure distortion terms. This is because different orders of nonlinearity ($y_n(t)$) may be correlated since the input basis functions in (1) ($w(t)$, $w_2(t), \ldots, w_N(t)$) are not orthogonal.

The orthogonalisation of the behavioural model is needed for the prediction of in-band distortion in wireless communication systems where the objective is to extract the uncorrelated part of the nonlinear output that is responsible for the degradation of system performance. The objective is thus to convert the nonlinear model in (1) into a model with orthogonal output components of the form

$$y(t) = \sum_{n=1}^{N} s_n(t) \tag{2}$$

where $s_n(t)$ represent the nth order orthogonal output component corresponding to $y_n(t)$. Orthogonality here is defined in the statistical sense as

$$R_{s_n s_m}(\tau) = E[s_n(t) s_m(t+\tau)] = 0$$

where R represents the cross-correlation function assuming that $s_n(t)$ and $s_m(t)$ are jointly wide-sense stationary (WSS) processes and E is the statistical expectation operator. As a result, the nonlinear output is expressed as a useful component $y_c(t)$ correlated with the input signal, an uncorrelated distortion component $y_d(t)$

$$y(t) = y_c(t) + y_d(t) \tag{3}$$

where using (2)

$$y_c(t) = s_1(t) \tag{4}$$

and

$$y_d(t) = \sum_{n=2}^{N} s_n(t) \tag{5}$$

The useful component of the signal $y_c(t)$ consists of the linearly amplified version of the input signal and the correlated part of the nonlinear terms (spectral regrowth terms represented by orders of the input signal). The correlated part of the distortion does not contribute to distortion noise but rather affects the signal level in a manner akin to gain saturation or enhancement of discrete tones. The uncorrelated component of the output y_d is additive distortion noise and affects system performance similar to additive white gaussian noise (AWGN). Thus, both correlated and uncorrelated components of the output affects the output SNR and BER in different ways.

In the following subsections we use Gram–Schmidt orthogonalisation [16] to formulate the output as a sum of uncorrelated terms for the power series model.

2.1 Gram–Schmidt orthogonalisation

Gram–Schmidt orthogonalisation is a mathematical procedure by which a set of non-orthogonal basis vectors is converted into an orthogonal set [16]. Let $\boldsymbol{w}_i$ be a set of basis vectors for a finite-dimensional vector space V then any vector $\boldsymbol{y} \in V$ can be written as a linear combination of these basis vectors.

$$\boldsymbol{y} = \sum_{n=1}^{N} a_n \boldsymbol{w}_n \tag{6}$$

The Gram–Schmidt orthogonalisation procedure can then be used to produce an orthogonal basis set $\boldsymbol{u}_n$ and hence the vector $\boldsymbol{y}$ is written in terms of the orthogonal basis as

$$\boldsymbol{y} = \sum_{n=1}^{N} b_n \boldsymbol{u}_n \tag{7}$$

where the orthogonal basis are produced as

$$\boldsymbol{u}_n = \boldsymbol{w}_n - \sum_{m=1}^{n-1} \alpha_{mn} \boldsymbol{u}_m \tag{8}$$

where

$$\alpha_{nm} = \frac{\langle \boldsymbol{w}_n, \boldsymbol{u}_m \rangle}{\|\boldsymbol{u}_m\|^2} \tag{9}$$

and the $\langle\rangle$ represents inner product. The new coefficients b_n are then found as

$$b_n = a_n - \sum_{m=n}^{N} b_m \alpha_{mn} \tag{10}$$

2.2 Orthogonalisation of the power series model

Instead of dealing with the general form of the power series model, we choose to deal with its envelop version since all the simulations done in this paper are at the envelop level. The envelope version of this model represents the nonlinear relationship between the complex envelopes of the input and the output waveforms denoted by $\tilde{w}(t)$ and $\tilde{y}(t)$ and can be developed as in [17]

$$\tilde{y}(t) = \sum_{\substack{n=1 \\ n \text{odd}}}^{N} b_n \tilde{w}_n(t) = \sum_{\substack{n=1 \\ n \text{odd}}}^{N} b_n |\tilde{w}(t)|^{n-1} \tilde{w}(t) \tag{11}$$

where b_n represents the envelope coefficients which are directly related to the coefficients a_n. The envelope coefficients b_n can be obtained by polynomial fitting of the measured AM–AM and AM–PM characteristics.

Applying the Gram–Schmidt procedure to the model in (11), the new set of orthogonal output components $\tilde{s}_n(t)$ in (2) can be obtained by replacing the input basis functions ($\tilde{w}_1(t)$, $\tilde{w}_2(t), \ldots, \tilde{w}_N(t)$) by a new set of orthogonal basis ($\tilde{u}_1(t)$, $\tilde{u}_2(t), \ldots, \tilde{u}_N(t)$).

Therefore, the orthogonal output components can be written as

$$\tilde{s}_n(t) = c_n \tilde{u}_n(t) \tag{12}$$

where c_n represents the orthogonal model coefficients and $u_n(t) = u^n(t)$ represents a new set of orthogonal inputs. Therefore, using Gram–Schmidt procedure, the new set of orthogonal inputs can be obtained as

$$\tilde{u}_n(t) = \tilde{w}_n(t) - \sum_{m=1}^{n-2} \alpha_{nm} \tilde{u}_m(t) \tag{13}$$

where α_{nm} is complex coefficient that represents the correlation between the $w_n(t)$ and $u_m(t)$ terms

$$\alpha_{nm} = \frac{E[\tilde{w}_n(t)\tilde{u}_m^*(t)]}{E[\tilde{u}_m(t)\tilde{u}_m^*(t)]} \tag{14}$$

and hence, the original set $\boldsymbol{w}_n(t)$ can then be written as a linear combination of the orthogonal set $u_n(t)$ as

$$\tilde{\boldsymbol{w}}_n(t) = \sum_{m=1}^{n} \alpha_{nm}\tilde{u}_m(t). \tag{15}$$

The new set of coefficients c_n that represents the new orthogonal nonlinear model is derived from the original model coefficients b_n as

$$c_n = \sum_{m=n}^{N} b_m \alpha_{mn} \tag{16}$$

Note that the new set of coefficients depends on the original envelop coefficients and the input signal power level represented by the correlation coefficient α_{mn}. Note that Gram–Schmidt orthogonalisation leads to a completely uncorrelated output terms regardless of the distribution of the input process. Special cases of this procedure can be developed when the distribution of the input process is known, such as using the Gaussian assumption as discussed in [4].

Considering a 5th order orthogonalised model, the orthogonal inputs are found using (13)

$$\begin{aligned} \tilde{u}_1(t) &= \tilde{w}_1(t) \\ \tilde{u}_3(t) &= \tilde{w}_3(t) - \alpha_{31}\tilde{u}_1(t) \\ \tilde{u}_5(t) &= \tilde{w}_5(t) - \alpha_{51}\tilde{u}_1(t) - \alpha_{53}\tilde{u}_3(t) \end{aligned} \tag{17}$$

and the new set of coefficients of the orthogonal model are obtained using (16) as

$$\begin{aligned} c_1 &= b_1 + \alpha_{31}b_3 + \alpha_{51}b_5 \\ c_3 &= b_3 + \alpha_{53}b_5 \\ c_5 &= b_5 \end{aligned} \tag{18}$$

where the correlation coefficients α_{mn} are found as in (14) as

$$\begin{aligned} \alpha_{31} &= \frac{E[\tilde{w}_3(t)\tilde{u}_1^*(t)]}{E[\tilde{u}_1(t)\tilde{u}_1^*(t)]} \\ \alpha_{51} &= \frac{E[\tilde{w}_5(t)\tilde{u}_1^*(t)]}{E[\tilde{u}_1(t)\tilde{u}_1^*(t)]} \\ \alpha_{53} &= \frac{E[\tilde{w}_5(t)\tilde{u}_3^*(t)]}{E[\tilde{u}_3(t)\tilde{u}_3^*(t)]} \end{aligned} \tag{19}$$

A 5th order orthogonalised power series model is shown in Fig. 1. Note that with the power series model, Gram–Schmidt orthogonalisation leads to new polynomial types when the distribution of the input process is known. For example, if the input process has a Gaussian distribution, the orthogonalisation procedure leads to Hermite polynomial representation of the nonlinear model. Another example is when the input process has a Poisson distribution where the orthogonalisation procedure leads to the Poisson–Charlier polynomial representation of the nonlinear model [18, 19].

2.3 Discussion

The above formulation enables treating the in-band portion of the uncorrelated distortion component $y_d(t)$ as additive

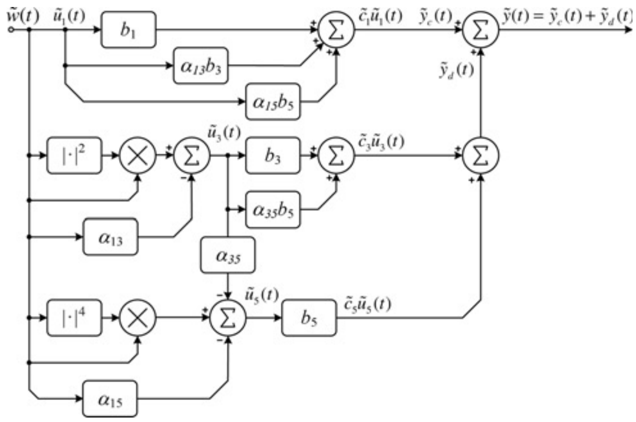

Fig. 1 *Orthogonal envelop nonlinear model with uncorrelated outputs of order 5*

noise component. Considering the power series model that represents the memoryless case, the uncorrelated in-band distortion causes the scattering of the constellation points of a digitally modulated signal similar to AWGN. On the other hand, when the uncorrelated term is removed (e.g. by linearisation), the resulting term is the correlated output which is a scaled version of the input signal by a complex factor. This scaling results in the rotation and scaling of the constellation points that is evident by considering the complex multiplicative factors $c1$ of the orthgonalised power series model. Therefore, from (12), the correlated output can be written as

$$\begin{aligned} \tilde{y}_c(t) = \tilde{s}_1(t) = c_1\tilde{u}_1(t) &= \sum_{m=n}^{N} b_m\alpha_{m1}\tilde{w}_1 \\ &= b_1\tilde{w}_1(t) + \sum_{m=3}^{N} b_m\alpha_{m1}\tilde{w}_1 \end{aligned} \tag{20}$$

which means that the terms other than the linear term $(b_1\tilde{w}_1)$ are what causes the rotation and scaling of the constellation points. In systems that employ automatic gain control (AGC), this scaling can be removed and hence the design of a predistorter on the basis of the removal of uncorrelated components results in optimum nonlinear distortion cancelation. Fig. 2*a* and *b* shows a constellation diagrams of a quadrature phase shift keying (QPSK) signal before and after nonlinear amplification, whereas Fig. 2*c* and *d* show the constellation of the correlated and uncorrelated components of the nonlinear output. These figures show that the constellation of the correlated output is a replica of the input with some rotation while the constellation of the uncorrelated output is shown to be similar to the case when AWGN is present.

3 Effective in-band distortion and SNDR

The objective now is to derive the output autocorrelation function and the output spectrum to characterise the effective in-band distortion. Using the orthogonal behavioural model, we can write the output autocorrelation function as the sum of uncorrelated components where the cross-correlation function of any two components is zero. Therefore, the output autocorrelation function can now be written using (3) as

$$R_{\tilde{y}\tilde{y}}(\tau) = R_{\tilde{y}_c\tilde{y}_c}(\tau) + R_{\tilde{y}_d\tilde{y}_d}(\tau) \tag{21}$$

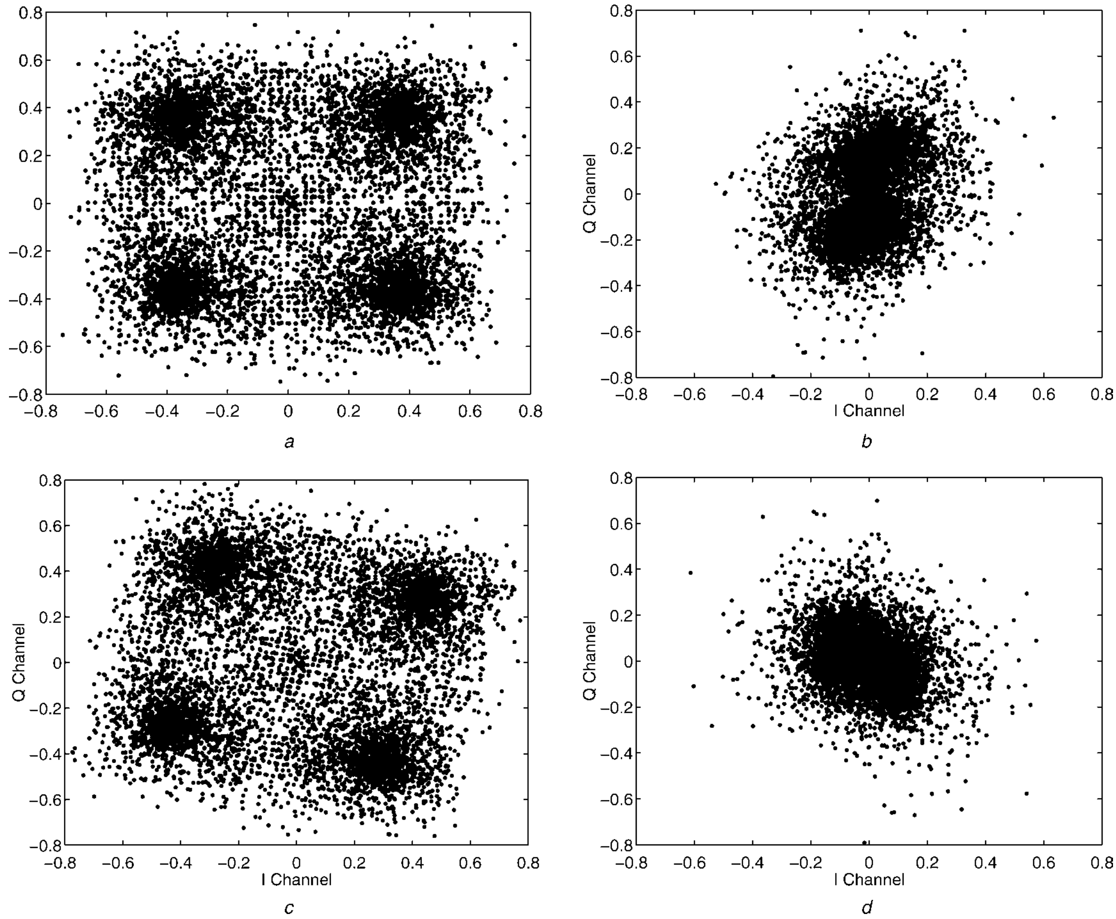

Fig. 2 *Constellation diagram of QPSK with pulse shaping*

a Input signal
b Output signal
c Correlated output
d Uncorrelated output

where the autocorrelation function is defined for a WSS stationary random process as

$$R_{\tilde{y}\tilde{y}}(\tau) = E[\tilde{y}(t)\tilde{y}^*(t+\tau)]$$

where the * represents complex conjugation. The Fourier transform of the autocorrelation function gives the power spectral density of the process by Wiener–Kenchin theorem [20]. Here, $R_{\tilde{y}_c\tilde{y}_c}(\tau)$ represents the autocorrelation function of the correlated component of the nonlinear output and $R_{\tilde{y}_d\tilde{y}_d}(\tau)$ represents the autocorrelation function of the uncorrelated component and can be expressed using (4) and (5) as

$$R_{\tilde{y}_c\tilde{y}_c}(\tau) = R_{\tilde{s}_1\tilde{s}_1}(\tau)$$

$$R_{\tilde{y}_d\tilde{y}_d}(\tau) = \sum_{n=2}^{N} R_{\tilde{s}_i\tilde{s}_i}(\tau) \tag{22}$$

where the autocorrelation function $R_{\tilde{y}_c\tilde{y}_c}(\tau)$ represents the undistorted output and $R_{\tilde{y}_d\tilde{y}_d}(\tau)$ represents distortion.

With the orthogonalisation of the nonlinear model, the output autocorrelation consists of the sum of the autocorrelation functions of the orthogonal components of the output. However and as pointed out in [7], this is not always true because even if $E[\tilde{s}_n(t)\tilde{s}_m^*(t)] = 0$ it is not necessarily that $E[\tilde{s}_n(t)\tilde{s}_m^*(t+\tau)] = 0$. Blachman [7] studied this case and showed that for a zero mean random process $\tilde{w}(t)$ a sufficient and necessary condition for the output autocorrelation function to be written as a sum of uncorrelated components is that the process $\tilde{w}(t)$ be a separable random process (has a separable distribution function) in the Nuttal sense [21]. This implies that the conditional mean $M = E[\tilde{w}(t+\tau)|\tilde{w}(t)]$ is a linear function of the form: $A(\tau)\tilde{w}(t) + B(\tau)$. Therefore, for any separable process, M can be written as [7]

$$\begin{aligned} M = E[\tilde{w}(t+\tau)|\tilde{w}(t)] &= \frac{E[\tilde{w}(t)\tilde{w}(t+\tau)]}{E[\tilde{w}^2(t)]}\tilde{w}(t) \\ &= \frac{R_{\tilde{w}\tilde{w}}(\tau)}{R_{\tilde{w}\tilde{w}(0)}}\tilde{w}(t) \end{aligned} \tag{23}$$

This means that the cross-correlation function of $\tilde{w}(t) = \tilde{w}_1(t)$ and its nth power ($\tilde{w}_n(t)$) can be written as

$$\begin{aligned} E[\tilde{w}_n(t)\tilde{w}_1(t+\tau)] &= E[E[\tilde{w}_1(t+\tau)\tilde{w}_n(t)|\tilde{w}_1(t)]] \\ &= E\left[\frac{E[\tilde{w}_1(t+\tau)\tilde{w}_1(t)]}{E[\tilde{w}_1^2(t)]}\tilde{w}_n(t)\tilde{w}_1(t)\right] \\ &= \frac{E[\tilde{w}_1(t+\tau)\tilde{w}_1(t)]}{E[\tilde{w}_1^2(t)]}E[\tilde{w}_n(t)\tilde{w}_1(t)] \\ &= \frac{R_{\tilde{w}_1\tilde{w}_1}(\tau)}{R_{\tilde{w}_1\tilde{w}_1}(0)}R_{\tilde{w}_n\tilde{w}_1}(0) \\ &= \alpha_{n1}R_{\tilde{w}_1\tilde{w}_1}(\tau) \\ &= \alpha_{n1}R_{\tilde{w}\tilde{w}}(\tau) \end{aligned} \tag{24}$$

The identity holds for separable random processes such as Gaussian processes as will be shown in the next section: however, a separable process does need to be Gaussian [7]. For CDMA signals, this condition can be proved using their statistical properties without assuming a Gaussian distribution.

To develop the relationship between the co-channel distortion and SNDR (and hence system BER), it is useful to express the above formulation in the frequency domain. The output power spectral density (PSD) is obtained from the Fourier transform of the autocorrelation function [20]. Therefore, the output PSD of $\tilde{y}(t)$ can be found by taking the Fourier transform of (21)

$$S_{\tilde{y}\tilde{y}}(f) = S_{\tilde{y}_c\tilde{y}_c}(f) + S_{\tilde{y}_d\tilde{y}_d}(f) \tag{25}$$

The output spectrum is therefore the sum of the spectra of the uncorrelated signal components of the orthogonal behavioural model. This partition is useful for the case in hand where the objective is to separate the uncorrelated output distortion from the useful or undistorted component and hence to estimate the effective in-band distortion. The effective in-band distortion can now be expressed in terms of the PSD's of the uncorrelated output components of (25) as

$$P_{\text{Inband}} = \int_{-B/2}^{B/2} S_{\tilde{y}_d\tilde{y}_d}(f)\,\mathrm{d}f \tag{26}$$

where B is the bandwidth of the input signal. The effective system SNDR is defined as the ratio of signal to total noise power including the effective in-band distortion power. It can be expressed in terms of the PSD's of the uncorrelated output components of (25) as

$$\text{SNDR} = \frac{\int_{-B/2}^{B/2} S_{\tilde{y}_c\tilde{y}_c}(f)\,\mathrm{d}f}{\int_{-B/2}^{B/2} S_{\tilde{y}_d\tilde{y}_d}(f)\,\mathrm{d}f + N_0 B} \tag{27}$$

Note that SNDR is a function of both the nonlinear distortion and the PSD of AWGN represented by N_0. The evaluation of the effective SNR is important to determine the system BER and the system noise figure. These parameters are usually estimated assuming a linear AWGN channel however nonlinear distortion increases the system BER for a fixed AWGN power. In the following, we apply the above analysis to the case where the input signal is a known such as the case of narrow band gaussian noise (NBGN) process and WCDMA signals.

3.1 Orthogonalised behavioural model with gaussian input

As an example, we apply the above analysis to a NBGN process which is a very widely used approximation to CDMA signals. A NBGN signal is represented by its autocorrelation function as as

$$R_{\tilde{w}\tilde{w}}(\tau) = E[\tilde{w}(t)\tilde{w}^*(t+\tau)] = N_0 \text{sinc}(B\tau) \tag{28}$$

where B is the bandwidth of the process and N_0 is the PSD of the white process before bandlimiting. Considering a 5th order orthogonalised power series model, the correlation coefficients α_{mn} can be evaluated using (14) and using the properties of Gaussian random processes [22]

$$\begin{aligned}
\alpha_{31} &= \frac{E[\tilde{w}_3(t)\tilde{u}_1^*(t)]}{E[\tilde{u}_1(t)\tilde{u}_1^*(t)]} = \frac{R_{\tilde{w}_3\tilde{u}_1}(0)}{R_{\tilde{u}_1\tilde{u}_1}(0)} = 2R_{\tilde{w}\tilde{w}}(0)\\
\alpha_{51} &= \frac{E[\tilde{w}_5(t)\tilde{u}_1^*(t)]}{E[\tilde{u}_1(t)\tilde{u}_1^*(t)]} = \frac{R_{\tilde{w}_5\tilde{u}_1}(0)}{R_{\tilde{u}_1\tilde{u}_1}(0)} = 6R^2_{\tilde{w}\tilde{w}}(0)\\
\alpha_{53} &= \frac{E[\tilde{w}_5(t)\tilde{u}_3^*(t)]}{E[\tilde{u}_3(t)\tilde{u}_3^*(t)]} = \frac{R_{\tilde{w}_5\tilde{u}_3}(0)}{R_{\tilde{u}_3\tilde{u}_3}(0)} = 6R_{\tilde{w}\tilde{w}}(0)
\end{aligned} \tag{29}$$

Let $A = R_{\tilde{w}\tilde{w}}(0)$ (which represents the average power of the input signal), the new set of coefficients of the orthogonal model is obtained using (18) as

$$\begin{aligned}
c_1 &= b_1 + 2Ab_3 + 6A^2 b_5\\
c_3 &= b_3 + 6Ab_5\\
c_5 &= b_5
\end{aligned}$$

the orthogonal inputs are evaluated using (19) as

$$\begin{aligned}
\tilde{u}_1(t) &= \tilde{w}_1(t)\\
\tilde{u}_3(t) &= \tilde{w}_3(t) - 2A\tilde{u}_1(t)\\
\tilde{u}_5(t) &= \tilde{w}_5(t) - 6A^2\tilde{u}_1(t) - 6A\tilde{u}_3(t)
\end{aligned}$$

and hence the orthogonal outputs can be evaluated as $\tilde{s}_i(t) = c_i\tilde{u}_i(t)$, $i = \{1, 3, 5\}$. It is easy to show that a Gaussian process is a separable random process because of the separability of the Gaussian distribution function and therefore the condition (23) holds. For example

$$\begin{aligned}
E[\tilde{s}_1(t+\tau)\tilde{s}_3(t)] &= E[c_1\tilde{u}_1(t+\tau)c_3\tilde{u}_3(t)]\\
&= c_1c_3E[\tilde{u}_1(t+\tau)\tilde{u}_3(t)]\\
&= c_1c_3E[\tilde{w}_1(t+\tau)\tilde{w}_3(t)]\\
&\quad - \alpha_{31}c_1c_3E[\tilde{w}_1(t+\tau)\tilde{w}_1(t)]
\end{aligned}$$

but using the properties of Gaussian processes [23]

$$E[\tilde{w}_1(t+\tau)\tilde{w}_3(t)] = 2R_{\tilde{w}\tilde{w}}(\tau)R_{\tilde{w}\tilde{w}}(0)$$

and hence

$$\begin{aligned}
E[\tilde{s}_1(t+\tau)\tilde{s}_3(t)] &= 2c_1c_3R_{\tilde{w}\tilde{w}}(\tau)R_{\tilde{w}\tilde{w}}(0)\\
&\quad - 2c_1c_3R_{\tilde{w}\tilde{w}}(0)R_{\tilde{w}\tilde{w}}(\tau)\\
&= 0.
\end{aligned} \tag{30}$$

The cross correlation between the other terms can be evaluated similarly. The autocorrelation of the correlated and uncorrelated components can now be evaluated using (22) and using the properties Gaussian random variables [23] as

$$\begin{aligned}
R_{\tilde{y}_c\tilde{y}_c}(\tau) &= R_{\tilde{s}_1\tilde{s}_1}(\tau) = |c_1|^2 R_{\tilde{u}_1\tilde{u}1}(\tau)\\
&= |b_1 + 2b_3A + 6b_5A^2|^2 R_{\tilde{w}\tilde{w}}(\tau)
\end{aligned} \tag{31}$$

$$\begin{aligned}
R_{\tilde{y}_d\tilde{y}_d}(\tau) &= R_{\tilde{s}_3\tilde{s}_3}(\tau) + R_{\tilde{s}_5\tilde{s}_5}(\tau)\\
&= |c_3|^2 R_{\tilde{u}_3\tilde{u}_3}(\tau) + |c_5|^2 R\tilde{u}_5\tilde{u}_5(\tau)
\end{aligned} \tag{32}$$

where

$$|c_3|^2 R_{\tilde{u}_3\tilde{u}_3}(\tau) = 2|b_3 + 6b_5A|^2 R^3_{\tilde{w}\tilde{w}}(\tau) \tag{33}$$

and

$$|c_5|^2 R_{\tilde{u}_5\tilde{u}_5}(\tau) = 12|b_5|^2 R^5_{\tilde{w}\tilde{w}}(\tau) \tag{34}$$

The output PSD can be found by taking the Fourier transform of (31) and (32). Thus, the output autocorrelation

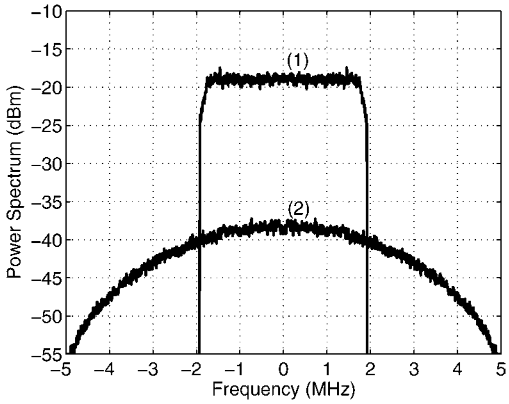

Fig. 3 *Output spectrum of NBGN process; (1) correlated output and (2) uncorrelated distortion*

function consists of uncorrelated components and the output spectrum is therefore the sum of the spectra of the individual components leading to a completely separable signal and distortion components. Fig. 3 shows the output spectrum of an NBGN process partitioned into correlated and uncorrelated components. It is clear that the shape of the uncorrelated distortion spectrum is completely different in shape from the correlated spectrum.

3.2 WCDMA signals

In this section, we consider the application of the orthogonal behavioural model using different examples of WCDMA signals. Simulated WCDMA are used as input to a 5th order power series model, and the output spectrum is computed from the orthogonalised model. Fig. 5 shows the total output spectrum and the uncorrelated distortion spectrum of three different customised forward-link WCDMA signals: forward-link 3 dedicated physical channel (DPCH) and 16 DPCH and a reverse-link 5DPCH signal. Note that the shape of the uncorrelated components depends on the signal and its statistics. Fig. 4 shows the probability density functions of the three signals. In the case of 3 DPCH and 5 DPCH shown in Fig. 5*a* and *b*, the uncorrelated distortion inside the main channel is below spectral regrowth in the adjacent channel. The case of 16 DPCH shown in Fig. 5*c* represents the worst case since it exhibits the highest peak-to-average ratio (PAR). The shape of the

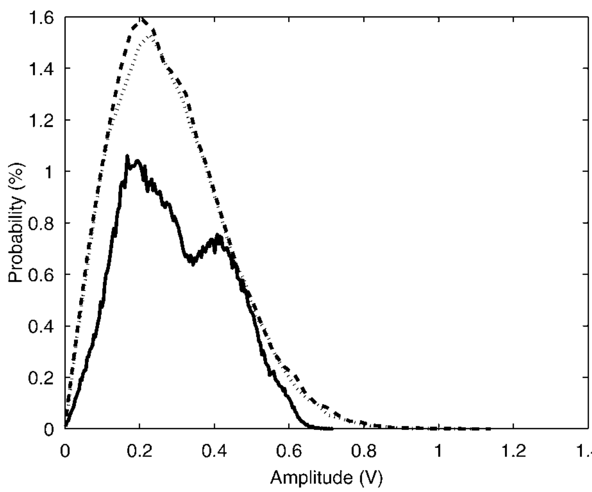

Fig. 4 *PDF of the signal envelope of the three WCDMA signals used in the simulations. Solid line, 5 DPCH; dashed line, 3 DPCH; dotted line, 16 DPCH*

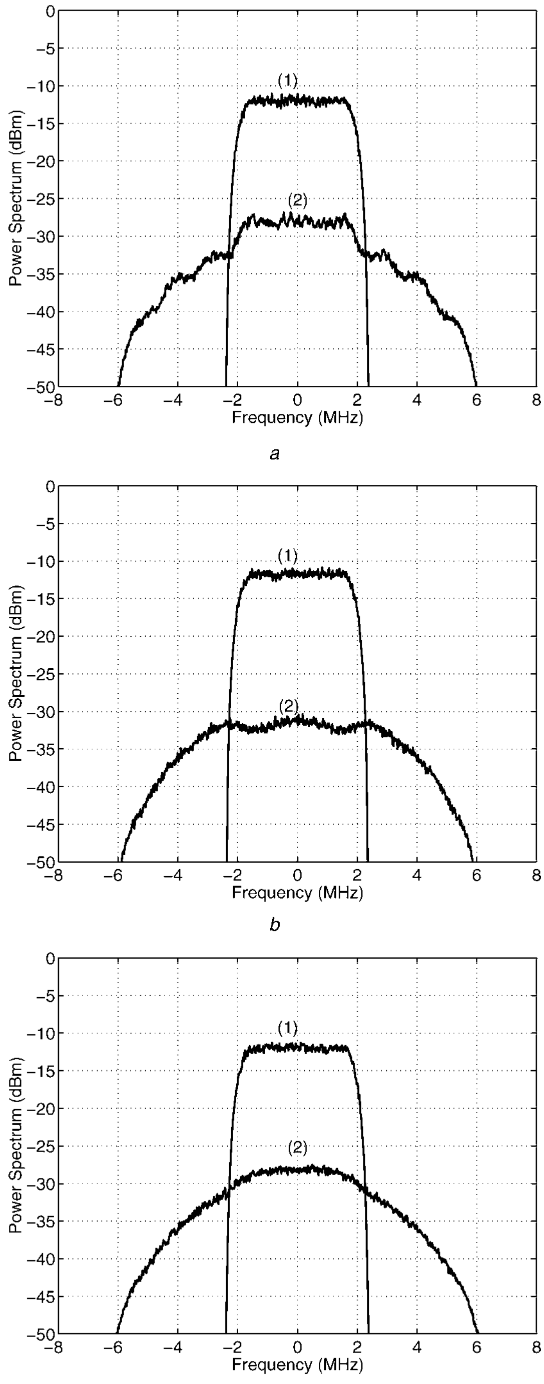

Fig. 5 *Output spectrum of a forward-link WCDMA signal. (1) correlated spectrum and (2) uncorrelated distortion*

a 3 DPCH
b 5 DPCH
c 16 DPCH

uncorrelated spectrum resembles that of Gaussian signals which is flat over the signal bandwidth because its distribution can be approximated by a Gaussian distribution by the central limit theorem.

4 Measurement and simulation results

The analytical evaluation of the uncorrelated distortion component obtained from the orthogonalisation procedure was verified by measurements done on WCDMA signals. The measurements presented here were done using Agilent 8510 vector network analyser (VNA), E4438C vector signal generator, E4445A spectrum analyser and 89600S vector signal analyser (VSA).

4.1 Behavioural model extraction

The amplifier considered here has a gain of 21 dB, an output 1 dB compression point of 11 dBm and an output third order intercept (OIP3) of 18 dBm all at 2 GHz. The coefficients of the envelope model of the amplifier were obtained by measuring the AM–AM and AM–PM characteristics of the power amplifier (PA) at 2 GHz. A polynomial of order 5 was fitted to the complex data using classical least squares polynomial fitting and a set of envelope coefficients (b_n) obtained. Table 1 shows the envelope coefficients of the envelope power series model developed to model the PA. The output spectrum was developed from the computed autocorrelation function using signal realisations of the forward-link and the reverse-link WCDMA signals which were generated according to the WCDMA standard.

4.2 Orthogonal model verification

Table 2 shows the corresponding orthogonal model coefficients for different WCDMA signals and at an input power of −10 and −5 dBm. The uncorrelated distortion spectrum was measured using feed-forward cancelation as described in [13]. The input signal is generated using Agilent ESG 4438C vector signal generator. Three forward-link WCDMA signals were generated: 3 DPCH, 5 DPCH and 16 DPCH, all using WCDMA standard. The resulting spectrum at the output of the feed-forward cancelation system consists of the uncorrelated distortion, and the effective in-band distortion is measured within the signal bandwidth using Agilent VSA. Fig. 6 shows the measured and simulated uncorrelated distortion spectra of the three forward-link WCDMA signal models: 3 DPCH, 5 DPCH, and 16 DPCH all, measured at $P_{in} = -10$ dBm which is the 1 dB compression point of the PA. The measured uncorrelated distortion spectrum was compared with simulated values using simulated spectrum and the orthogonal behavioural model. The simulated spectra show a good agreement with the measured spectra in terms of the power levels and the shape. Fig. 7 shows the measured and simulated in-band distortion as a function of the output power, all measured in a bandwidth of 3.84 MHz of WCDMA signals. The difference between the measured and simulated values at low power levels is because of the finite cancelation that the feed-forward approach provides.

The predicted SNDR was verified using direct vector signal analyzer (VSA) measurements. Fig 8 shows a good agreement

Table 1: Envelope power series coefficients

b_1	$3.7576 + j10.7590$
b_3	$-89.8974 - j83.1693$
b_5	$495.24 + j330.57$

Table 2: Orthogonal power series coefficients for various WCDMA signals

P_{in}, dBm		3-DPCH	5 DPCH	16 DPCH
−10				
	c_1	$2.9166 + j9.9608$	$3.0392 + j10.0826$	$2.9249 + j9.9689$
	c_3	$-74.9131 - j73.1674$	$-80.8735 - j77.1459$	$-74.9111 - j73.166$
	c_5	$495.24 + j330.57$	$495.24 + j330.57$	$495.24 + j330.57$
−5				
	c_1	$1.6327 + j8.5917$	$1.7969 + j8.8278$	$1.6480 + j8.6100$
	c_3	$-42.5131 - j51.5405$	$-61.3613 - j64.1216$	$-42.5067 - j51.5362$
	c_5	$495.24 + j330.57$	$495.24 + j330.57$	$495.24 + j330.57$

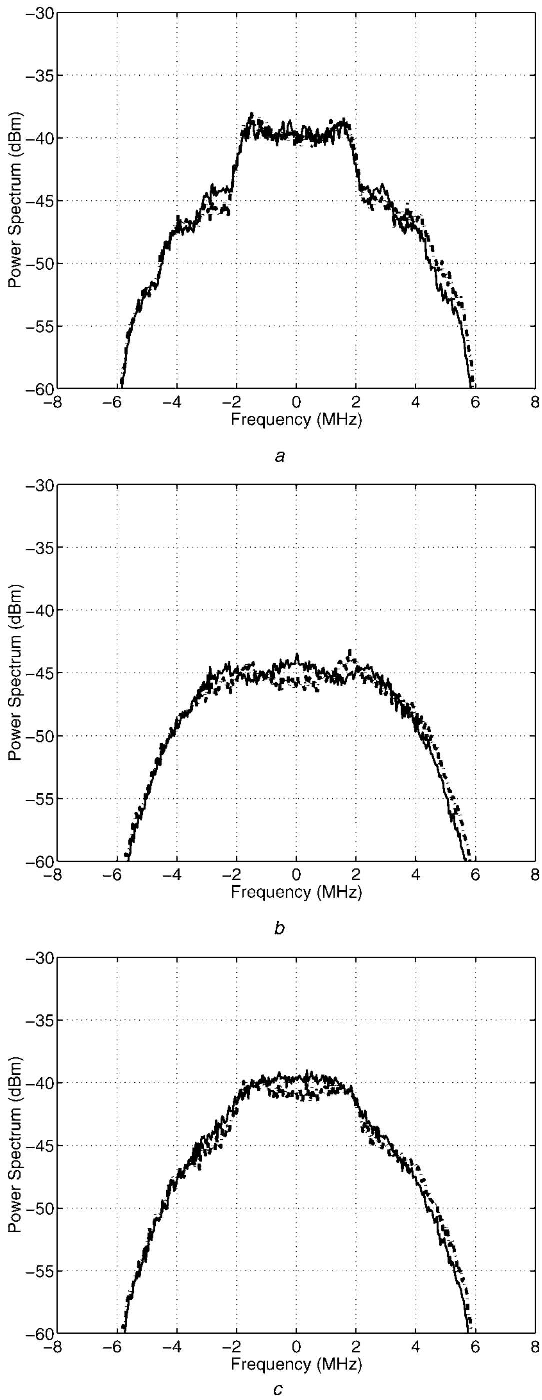

Fig. 6 *Uncorrelated distortion spectrum of a forward-link WCDMA signal. Dashed line, measured signal; solid line, simulated signal*
a 3 DPCH
b 5 DPCH
c 16 DPCH

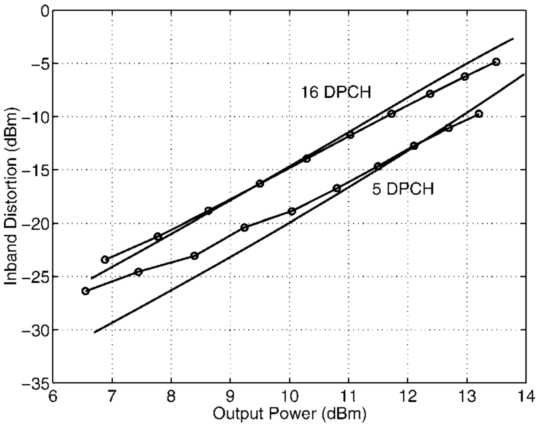

Fig. 7 *In-band distortion against output power. Lines with dot, measured power; solid lines, simulated power*

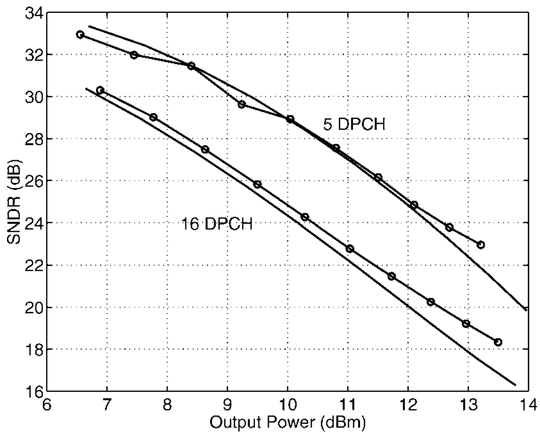

Fig. 8 *SNDR against output power. Lines with dot, measured power solid lines, simulated*

between the measured and the simulated values of SNDR. The signal to additive white Gaussian noise ratio was fixed at 30 dB in the simulations which is the value measured by the VSA when no nonlinear distortion is present.

5 Conclusion

We have developed a procedure that enables the effective in-band distortion in RF front ends to be identified and estimated. The procedure is based on the orthogonalisation of the nonlinear model using Gram–Schmidt orthogonalisation. The orthogonalisation was done for the power series model. The analytical formulation presented here enables the SNDR to be directly related to nonlinear distortion for both memoryless system and systems with memory. The uncorrelated co-channel distortion was obtained from the output spectrum estimated using signal realisations and measured nonlinear characteristics, which means that system metrics and in-band distortion can be estimated directly and accurately from simple measurements.

6 Acknowledgments

This material is based upon work supported/supported in part by the US Army Research Laboratory and the US Army Research Office under grant number W911NF-05-1-0337 and under the William J. Pratt Assistant Professorship.

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