Electro-Thermal Passive Intermodulation Distortion in Microwave Attenuators

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***Abstract*—This paper explores the generation of passive intermodulation distortion products through the self heating of resistive elements in a microwave attenuator. A compact electrothermal resistor model requiring only two physical parameters is developed for platinum resistors and applied to a resistive attenuator Pi network to predict electrothermal PIM. The electrothermal model is verified by comparing measured and predicted results when the attenuator is excited by a two tone signal at 400MHz with tone spacing from 1 to 100 Hz.**

I. Introduction

Passive Intermodulation Distortion (PIM) is an important problem in any high power radio frequency or microwave system, including satellite and cellular systems. PIM, like its active counterpart, is produced by the introduction of a nonlinear component to a system. The nonlinearity in this component will generate harmonics if a single tone is applied to the system. In addition, if two tones or more are applied to the system, intermodulation products will arise that can fall in the pass band of the system and become the limiting factor in the dynamic range of that system.

Although several PIM mechanisms such as ferromagnetic materials, metal-dielectric-metal contacts, cracks and holes in metal structures, dirty materials, and dirty contacts have been identified as possible sources of PIM [1,2], they are difficult to isolate experimentally and reproducibly, making it all but impossible to compare theory and experimental data. Models to predict these phenomena are scarce due to the difficulty in experimental isolation of PIM sources. Thermal processes are also mentioned in [1] as a possible source of PIM. Unlike other sources of PIM that are difficult to separate, thermal process generation of PIM will be shown here to be reproducibly experimentally isolatable.

This paper describes the theory behind electrothermally induced passive intermodulation distortion, explores compact models to accurately simulate electrothermal PIM, and demonstrates measurements for model identification and validation. The compact model introduced can be produced based on only two physical parameters, thermal resistance and thermal capacity, and accurately models PIM production due to thermal processes. To physically verify the model, a platinum resistor attenuator Pi network was built and tested by applying a two tone signal at 400MHz with tone spacing swept from 1 to 100 Hz. PIM was successfully measured and simulated, with both results in very good agreement.

II. Electrothermal PIM Theory

When a single carrier is applied to a resistive element, the electrothermal processes responsible for modulating the device resistance will average out to provide a stable resistance because the thermal capacity cannot react quickly enough to the high frequency signal to significantly heat or cool the resistive material. The situation changes drastically when two or more signals are applied to the device. The combination of two carriers results in a time varying signal envelope. The instantaneous power of this signal varies sinusoidally at the beat frequency of the two tone input to the device. If the beat frequency is within the low pass filter created by the thermal resistance and thermal capacity of the element, periodic heating and cooling of the element will occur. The heating and cooling of the resistive element will function as a passive mixer, facilitating intermodulation distortion generation.

The physical origin of thermal process induced passive intermodulation distortion is the thermoresistance effect, which states specific resistivity (Ω m) of a material can be expressed as a function of temperature in the relationship [3]

$$\rho(T) = \rho_o(1 + \alpha T + \beta T^2 + ...) . \tag{1}$$

In a resistive element, self heating, or the heating of a resistive element due to the power dissipated in that element, is the driving force behind the thermoresistance effect. Self heating is commonly quantified in terms of the thermal resistance ($K°/W$) according to the equation

$$R_{th} = \frac{\Delta T}{P} = \frac{\Delta T}{I^2 R} . \tag{2}$$

Although steady state temperature can be calculated using the power dissipated in the device and its thermal resistance, the instantaneous temperature can not be calculated using these parameters. The reason for this is that heat transfer does not occur instantaneously, which is shown in the parabolic heat transfer equation given by

$$\nabla \cdot \left(\frac{\nabla T}{R_{th}} \right) + Q - \rho C_P \frac{\delta T}{\delta t} = 0 . \tag{3}$$

Here the generated heat per unit volume (W /m^{-3}) is defined as

$$Q = J^2 \rho , \tag{4}$$

where J is the current density vector in A/ m^{-2}.

The finite time needed to transfer heat is captured by the thermal capacity, the combination of the ability of the material to store heat by raising its temperature and to conduct heat to its surrounding environment at a given rate, which could include heat radiation or conduction. The thermal capacity can be expressed as

$$C_v = \left(\frac{\delta Q}{dT}\right)_{volume=const} = T\left(\frac{\partial S}{\partial T}\right)_{volume=const} . \qquad (5)$$

Here Q is heat and S is the entropy of the system.

While not the direct cause of PIM in the thermal process, thermal capacity still has a substantial effect on passive intermodulation distortion. The combination of the thermal resistance and capacitance forms a low pass filter in the thermal domain. Heat flow is a power process, so the slope of the low pass filter is not twenty decibels per decade as in voltage and current based analog filters. The thermal capacity is a function of multiple processes including heat storage, conduction, and radiation, which leads to a filter slope of approximately ten decibels per decade.

III. Compact Thermal Model

In general, thermal models are used to determine the actual operational temperature of electrical components. This information is used to correct the simulated I-V characteristics of a device and determine corrected maximum device operating conditions. The two most prominent methods for electrothermal simulation are detailed numerical simulation and compact thermal models [3]. The detailed numerical methods include finite difference, finite element, and boundary element methods. These methods provide the entire temperature distribution of a component but are very computationally intensive. To enable simulation on a large scale, a compact thermal model, usually composed of a small electrical network, is preferred because it is computationally efficient and simple to model. The compact model of the self-heating effect in a resistive element is simply a RC filter with the power dissipated providing a current into the thermal resistance and thermal capacity of the device, as shown in Fig. 1.

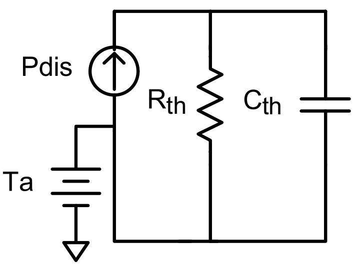

Fig. 1: Compact Thermal Model.

The basic compact model can not capture the time constant of the thermal process accurately because it is based on the assumption that the thermal process is exponential. Significant deviations from exponential behavior are commonly seen [4], but electrothermal models still commonly use an exponential fit to give the time constant of the compact thermal model. Although this model accurately gives final values of temperature and approximates real circuit time constants, it fails to accurately model the frequency response of a real thermal model in electrical simulators. The step response to a current step test for a platinum resistor, the compact electro-thermal model response, and the frequency corrected electro-thermal model response are shown in Fig. 2. The compact thermal model is not able to accurately model the thermal transient while the frequency corrected model approximates the step response much more effectively.

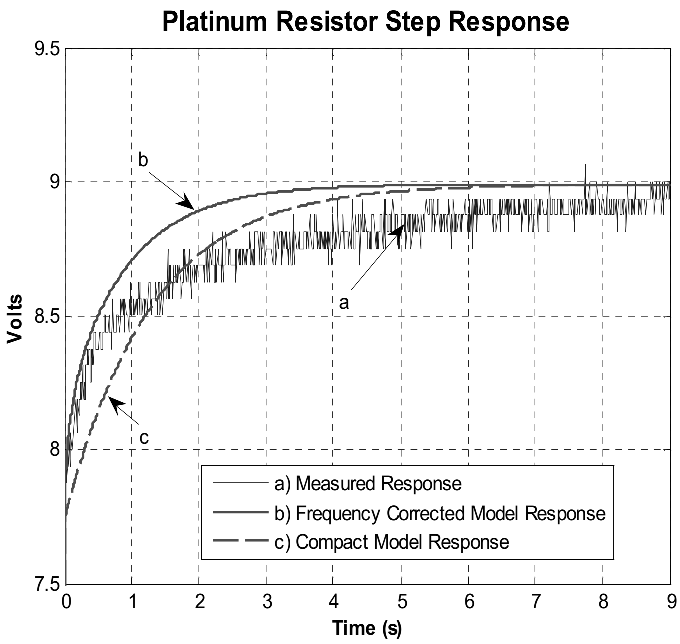

Fig. 2: a) Measured current step test, b) FCCM, and c) CM.

IV. Frequency Corrected Compact Thermal Model

In order to generate the compact thermal model for a device, both the thermal resistance and capacity must be determined for the component with any heat sinking taken into account, as it will be employed in the final design. Monitoring the element voltage response to a current step input is the most convenient test to determine both the thermal resistance and capacity for a device in its operational environment. The thermal resistance is the difference in the final value of resistance for a given power step minus the ambient resistance, normalized to one watt. The thermal capacity can be found by fitting the power response curve to the equation [4]

$$P(T) = P_0\left(1 - e^{-t/R_{th}C}\right) . \qquad (6)$$

In circuit simulators, the thermal capacity is modeled by a capacitor which yields a twenty decibel per decade filter slope. A circuit must be synthesized to yield the correct filter slope and time constants while not greatly increasing the computational efficiency of the model. The required circuit can be synthesized by using a partial fraction expansion on the desired admittance transfer function of the thermal response in the form of a Foster expansion according to the formula [5]

$$Y(s) = sk'_\infty + k'_0 + \sum_{i=1}^{n} \frac{sk_i}{s+\sigma_i} . \quad (7)$$

The desired admittance transfer function itself can be computed from the original physical parameters, the thermal resistance and thermal capacity. By fitting the step response of a single device in its thermal environment, the dominant pole can be found using the power response equation and

$$f_{3dB} = \frac{1}{2\pi R_{th} C} . \quad (8)$$

The rest of the transfer function can be obtained by alternating poles and zeroes at frequencies given by

$$f_{ci} = f_{co} \left(\frac{f_{c(N-1)}}{f_{co}} \right)^{\frac{i}{2N}} , \quad (9)$$

where f_{co} is the lowest frequency of interest, $f_{c(N-1)}$ is the highest frequency of interest, N is the order of the approximation, and f_{ci} is the crossover frequency where the estimated response crosses the intended response. The necessary poles and zeroes can be found according to the equations,

$$f_{pk} = -f_{c2k} \left(\frac{f_{c(N-1)}}{f_{co}} \right)^{\frac{2k-\alpha}{2N}} \quad (10)$$

$$f_{zk} = -f_{c2k} \left(\frac{f_{c(N-1)}}{f_{co}} \right)^{\frac{2k+\alpha}{2N}} \quad (11)$$

where α is the desired slope of the filter. A detailed description can be found in [6].

The admittance transfer function of the desired filter can be expressed as

$$\frac{Y(s)}{s} = \frac{(s+p_1)(s+p_2)...(s+p_N)(s+p_{N+1})}{s(s+z_1)(s+z_2)...(s+z_N)} , \quad (12)$$

where $Y(s)/s$ can now be foster expanded to yield

$$Y(s) = g\left(k'_0 + \sum_{i=1}^{n} \frac{sk'_i}{s+\sigma_i} \right) , \quad (13)$$

and the network can be synthesized according to [5]

$$R_i = \frac{1}{k'_i}, C_i = \frac{k'_i}{\sigma_i} . \quad (14)$$

The required network will be a resistor in parallel with N series RC branches. According to the thermal resistance equation

$$P \times R_{th} = \Delta T , \quad (15)$$

the final value of resistance in the model must be the thermal resistance to accurately model the temperature. This implies that the foster expansion must be multiplied by the gain factor

$$g = \frac{1}{R_{th} k'_o} . \quad (16)$$

To guarantee that the filter magnitude continues to roll off, a capacitor generating a pole at the end of the approximated filter response must be added in parallel with the rest of the model. Due to constraints on the time constants of the circuit, this capacitor is no longer the thermal capacity of the original model. It must be selected to not significantly add to the time constant of the filter. In this model, the thermal capacity was computed according to

$$C_\infty = \frac{k_1 R_{th}}{z_N} , \quad (17)$$

where k_1 is a constant that shifts the pole to the end of the approximation response. The final form of the electrothermal model is shown in Fig. 3.

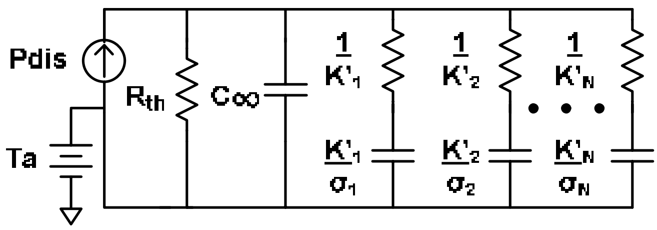

Fig. 3: $N+1$ order frequency corrected compact electrothermal model.

V. Measurements

A resistive attenuator Pi network was built and used to evaluate electrothermal PIM in a RF system. Spacing frequencies of 1 Hz to 100 Hz were measured. For each power level, the two tone spacing was swept logarithmically, ten points per decade. The through signal was measured from the 32.5 dB attenuator, with PIM being up to 58 dBc down from the carrier signal level. PIM magnitude was observed to decrease with an approximate 10 dB per decade slope as the carrier frequency separation was swept in the measurement range as shown in Fig. 4. Simulation results produced a very close replication of the observed PIM magnitude over the measurement range with respect to tone spacing, also pictured in Fig. 4, with no significant deviations. Fig. 5 shows the electrothermal induced spectrum.

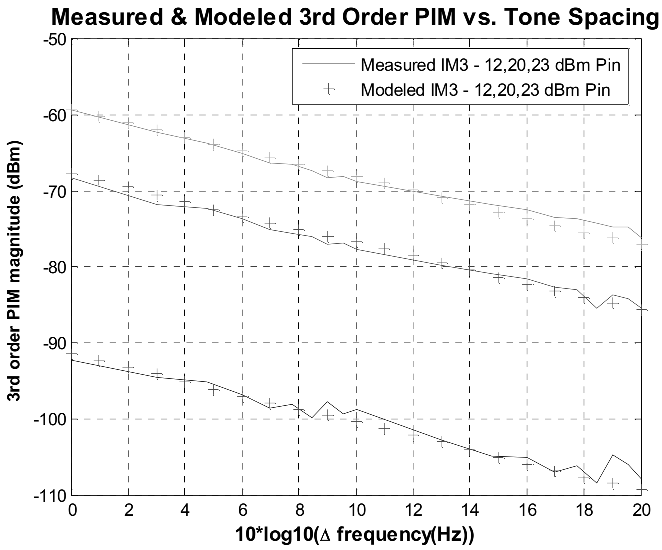

Fig. 4: Measured and modeled 3rd order PIM versus tone spacing.

Two different types of attenuators were built for comparison. A platinum resistance temperature detector based attenuator was built because of its high, almost linear temperature coefficient of resistance (TCR), 3850 ppm, and complete lack of any ferromagnetic materials. A nickel chromium based attenuator was also built because of its extremely low TCR of 5 ppm, which averages over the operational temperature range to almost zero parts per million. PIM from the platinum attenuator was measurable, but due to its low TCR, no PIM products in the measurement range were observable for the nickel chromium based attenuator.

The magnitude of electrothermal generated PIM from an element is strongly linked to the TCR of that element due to thermal baseband resistive mixing with the input signal. Lower TCR means lower thermal resistance, which reduces the thermal resistance variation and thus the PIM products. It follows that electrothermal PIM is minimized by using low TCR components in circuit design.

The intermodulation distortion magnitude is dependant on tone spacing, as is seen in Fig. 4. The closer the tones the longer the time period for heating and cooling of the material, leading to wider resistivity swings. As shown in Fig. 4, the wider the tone spacing of the carriers, the lower the magnitude of the electro-thermal PIM products. The reduction of PIM magnitude with wider tone spacing shows the thermal time constant of the material has a substantial effect on PIM magnitude. Minimization of PIM products can also be achieved through use of materials with large thermal time constants in circuit design.

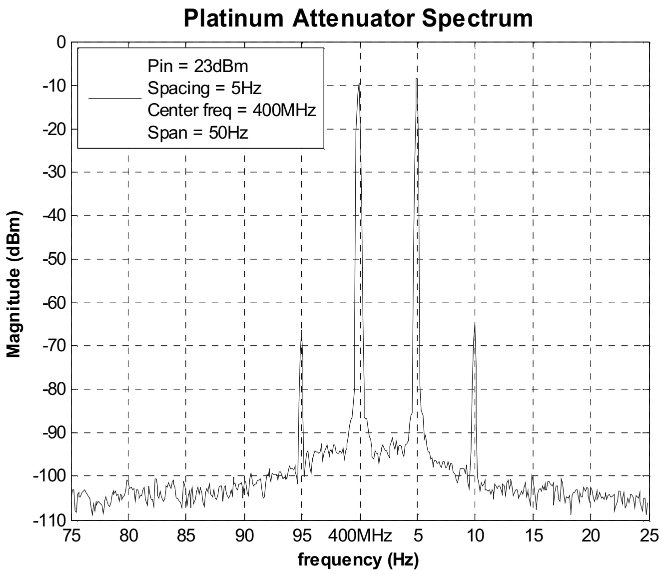

Fig. 5: Platinum attenuator frequency spectrum at 400 MHz center frequency and tone spacing of 5 Hz.

VI. CONCLUSIONS

An electrothermal model was presented for predicting electrothermal PIM in resistive elements. The model is computationally efficient and provides the thermal frequency domain accuracy required to accurately model electrothermal distortion frequency response. To validate the model, resistive attenuator Pi networks were built from platinum and nickel chromium discrete elements. The attenuators were excited by a two tone signal with swept tone spacing and the through response was measured. The frequency corrected electrothermal model simulation results and the measured results matched with very good agreement. Electrothermal PIM is governed by material properties and signal characteristics. To minimize PIM, the power envelope of the input signal should have minimal frequency content within the material thermal filter. Electrothermal PIM is also closely related to device TCR, imposing the need to design with low TCR components when high linearity is desired.

REFERENCES

[1] He Huan, Fu Wen-bin, "On Passive Intermodulation at Microwave Frequencies", *Proc. Asia-Pacific Conf. on Environmental Electromagnetics*, pp. 422 – 425, 4-7 Nov. 2003.

[2] Carlos Vicente and Hans L. Hartnagel, "Passive-Intermodulation Analysis Between Rough Rectangular Waveguide Flanges", *IEEE Trans. on Microwave Theory and Tech.*, vol. 53, no. 8, pp. 2515 – 2525, Aug. 2005.

[3] Bechtold, Rudnyi, and Korvink, "Dynamic Electro-Thermal Simulation of Microsystems-a Review", Micromechanics and Microengineering, vol. 15, no. 11, pp. R17-R31, Oct. 2005.

[4] Szepessy and Zoltan, "Thermal Dynamic Model of Precision Wire-Wound Resistors", IEEE Transactions on Instrumentation and Measurement, vol. 51, no. 5, pp. 930-934, Oct. 2002.

[5] G. C. Temes and J. W. LaPatra, *Introduction to Circuit Synthesis and Design*, McGraw-Hill, Inc. New York, 1977.

[6] K. Kundert, "Modeling Dielectric Absorption in Capacitors", Available from www.designers-guide.org.