Theoretical Analysis and Characterization of the Tunable Matching Networks in Low Noise Amplifiers

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***Abstract*—This paper provides the analytical background for the tunability in low noise amplifiers achieved by employing a π-network where the series matching capacitor is assumed to be variable. The theoretical outcomes suggest that the choice of the tunable capacitor value will eventually result in a tradeoff between the noise figure (NF) and the input return ratio (S_{11}). By pursuing a minimized noise figure approach, equations relating the value of the tunable capacitor at different frequencies with circuit parameters are derived and confirmed by the simulation results. Finally, it is demonstrated that to accomplish the design targets for NF and S_{11} over the tunable band, variable capacitors having quality factors greater than 10 should be utilized.**

Keywords- Low noise amplifier (LNA), matching network, noise matching, tunable capacitor, tunable LNA.

I. INTRODUCTION

Low noise amplifiers (LNAs) are an indispensable component of receiver structures for wireless communications. Their task is to ensure that the overall noise figure of the receiver can be kept minimal. The noise figure formula for cascaded stages reveals that this goal could be managed if the first stage of the total structure exhibits low noise figure and high gain at the operation frequency. In order to realize both specifications, a variety of design procedures have been developed [1]-[4]. Besides, the narrowband nature of communication standards necessitated the development of matching networks with high quality factors for the LNA designs.

Nevertheless, with the current progress in the wireless technologies, the number of communication standards increased swiftly. Narrowband designs require a receiver structure for each standard and this resulted in a plethora of employed hardware and in turn excessive power consumption which is highly undesired in integrated circuit design. Thus, a growing interest in multiband receiver topologies has emerged [5]. One option to realize multiband LNAs for this kind of receivers is utilizing tunable reactive components in the matching networks so that different frequencies can be covered by the same structure.

Employing a tunable component can be both done in the input and output matching network of the LNA. However, due to the nonlinear nature of the tunable components, designers preferred using them in the output loads of the low noise amplifiers [5]-[8]. Relying on the isolation between the output and input matching networks via the cascode topology, a tunable component at the load can assure multiband operation. However, these designs are either based on broadband input matching which in turn leads to unsatisfactory noise performance or they are operating only at two channels such that there is no tunability aspect in their final topology.

The authors of this work assume that rapid technological developments in the tunable reactive components will yield on-chip variable capacitors and inductors having better noise and linearity characteristics such that they can be utilized in the input matching without significant performance degradations. In regard to this consideration, a novel design methodology has been developed using heterojunction (SiGe) bipolar transistors which allows the designers to employ a series capacitor via the π-matching network at the base of the common emitter transistor in a cascode topology. This series capacitor can provide a tunable band between 3.5 GHz and 6.5 GHz if it would be made variable [9], [10]. In this paper, a theoretical analysis of the pursued strategy is given accompanied by a basic characterization of the tunable capacitor using the design constraints.

II. THEORY OF TUNABILITY USING THE SERIES MATCHING CAPACITOR AT THE INPUT

Fig. 1 depicts the decomposed version of the π-matching network that was used in the tunable LNA design. This version of the classical matching topology will be utilized to show that by using a single series matching capacitor both the noise and input impedance matching can be fulfilled.

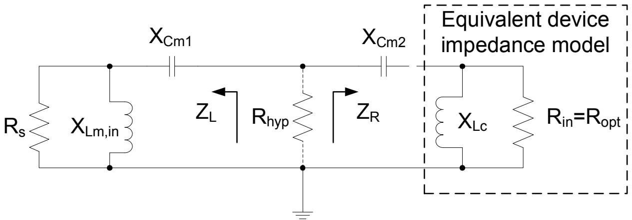

Figure 1. Decomposed π-matching network

Using the design procedure that was introduced in the previous work, the real parts of the amplifier input impedance R_{IN} and optimum noise impedance of the LNA -the impedance that will result in minimum noise figure (NF_{min}) operation-, R_{OPT}, have been equalized to the source resistance R_S through the emitter inductor L_e and extra base-emitter capacitor C_{ex} [9],

[10]. Also, the imaginary parts of the input impedance X_{IN} and optimum noise impedance X_{OPT} (referred to the device side) have been brought to the same value which is the reactance of the shunt bias inductor L_c. The matching network depicted in Fig. 1, takes into account these design choices by representing the equivalent input and noise impedances seen at the base of Q_1 in Fig. 2 as a combination of the inductor reactance X_{Lc} and the equalized resistances R_{IN} and R_{OPT} that are functions of the transistor parameters, the device current, L_e and C_{ex}.

At this step, one should focus on the middle of the two capacitors C_{m1} and C_{m2} which yield together the series capacitor $C_{m,in}$ in Fig. 2. It is possible to determine the equivalent impedances Z_L and Z_R when looking into the source and the device respectively (Fig. 1), as shown in (1) and (2).

$$Z_L = \frac{R_s\left(X_{Lm,in}\right)^2}{R_s^2 + X_{Lm,in}^2} + j\left(X_{Cm1} + \frac{X_{Lm,in}R_s^2}{R_s^2 + X_{Lm,in}^2}\right) \tag{1}$$

$$Z_R = \frac{R_{opt}\left(X_{Lc}\right)^2}{R_{opt}^2 + X_{Lc}^2} + j\left(X_{Cm2} + \frac{X_{Lc}R_{opt}^2}{R_{opt}^2 + X_{Lc}^2}\right) \tag{2}$$

By choosing the inductors $L_{m,in}$ and L_c such that their associated impedances become sufficiently smaller than the resistances R_S and R_{OPT}; the impedance expressions in (1) and (2) can be further simplified.

$$Z_L = \frac{\left(X_{Lm,in}\right)^2}{R_s} + j\left(X_{Cm1} + X_{Lm,in}\right) \tag{3}$$

$$Z_R = \frac{\left(X_{Lc}\right)^2}{R_{opt}} + j\left(X_{Cm2} + X_{Lc}\right) \tag{4}$$

It should be remembered that these resistances have to match with the hypothetical resistance R_{hyp} forcing the complex parts of Z_L and Z_R to be equal to zero. This condition obligates the following constraints on the inductor and capacitor impedances.

$$X_{Cm1} = -X_{Lm,in} \tag{5}$$

$$X_{Cm2} = -X_{Lc} \tag{6}$$

Similarly, the real parts need to be equal to R_{hyp}.

$$R_{hyp} = \frac{\left(X_{Lm,in}\right)^2}{R_s} \tag{7}$$

$$R_{hyp} = \frac{\left(X_{Lc}\right)^2}{R_{opt}} \tag{8}$$

It is possible to combine (7) and (8) as given in (9) :

$$\left(X_{Lm,in}\right)^2 = \left(X_{Lc}\right)^2 \frac{R_s}{R_{opt}} \tag{9}$$

Next, using (5), (9) can be modified to yield (10):

$$X_{Cm1} = -X_{Lc}\sqrt{\frac{R_s}{R_{opt}}} \tag{10}$$

By replacing the impedances with their corresponding expressions in terms of the capacitances and inductances, it is possible to rearrange (10) to (11):

$$C_{m1} = \frac{1}{\omega^2 L_c}\sqrt{\frac{R_{opt}}{R_s}} \tag{11}$$

The value of C_{m2} can be calculated by applying a similar operation on (6):

$$C_{m2} = \frac{1}{\omega^2 L_c} \tag{12}$$

Finally, the two capacitors C_{m1} and C_{m2} are in series so they can be combined together to give a single capacitor, $C_{m,in}$.

$$C_{m,in} = \frac{C_{m1}C_{m2}}{C_{m1} + C_{m2}} = \frac{1}{\omega^2 L_c}\frac{\sqrt{R_{opt}}}{\sqrt{R_{opt}} + \sqrt{R_s}} \tag{13}$$

While deriving (13), R_{OPT} has been given priority in impedance calculations over R_{IN} since in a low noise amplifier design; the noise consideration generally overweighs to the input return ratio specifications. It is actually possible to retain the equality between R_{OPT}, R_{IN} and R_S, however that will necessitate other reactive elements to be tunable (such as the emitter inductor L_e) leading to performance drops due to the nonlinear characteristics of those components. The choice of preserving low noise characteristics of the LNA will lead to deteriorations in input power matching over the tunable band and actually determine the feasible tunability range.

Until this point, the tunability of the output matching network has not been mentioned. In fact, tuning the circuit in the input matching network is not sufficient for a sustainable gain unless the output matching network is ensured to have a reasonable return ratio, i.e. S_{22}<-10 dB. Thus, a secondary tunable element is obligatory to use in the output matching network. Due to the isolation between the input and output in a cascode topology, the output matching affects neither the input referred noise nor the input matching conditions. Thus, the only constraint on the selection of the matching structure is to provide complex conjugate of the output impedance. By employing a matching network similar to the one at the input where again a series matching capacitor, $C_{m,out}$, is made use of, the desired results can be acquired easily [9].

In order to verify (13), the LNA topology given in Fig. 2 is selected, where the tunability phenomenon has been observed. In this circuit, only the reactive matching components are allowed to be ideal to demonstrate the validity of the derived equations. Other components have been chosen from the IBM 0.18 µm (7HP) design kit. First of all, the assumption that the impedances of $L_{m,in}$ and L_c are sufficiently smaller compared to R_S and R_{OPT} respectively, is checked. For the circuit simulation, the transistors of the cascode structure have been sized to an emitter length of 19.2 µm having a drain current of 2.55 mA. The emitter inductor L_e and the extra base-emitter capacitance C_{ex} are set to 750 pH and 120 fF respectively, whereas the bias inductor L_c has been chosen to be 1.08 nH. Then, without including $L_{m,in}$ and $C_{m,in}$, the optimum noise resistance R_{OPT} is simulated. The resulting graph shown in Fig. 3 indicates that R_{OPT} is much larger than the impedance of L_c for frequencies less than 7 GHz. Next, $L_{m,in}$ value is determined to get the design targets of $NF=NF_{min}$ and $S_{11}<-10$ dB. The resulting value of 670 pH leads to an impedance about the half of R_S (50 Ω) at 6.5 GHz, which is permissible. Subsequently, using the values of L_c, R_{OPT} and R_S the required value of $C_{m,in}$ has been calculated for different frequencies using (13). In the last step, the complete LNA circuit originally designed at 5 GHz with the component values given above for simultaneous input power and noise matching has been modified via changing $C_{m,in}$ and $C_{m,out}$ such that the noise matching can still be ensured at all frequencies. The sets of theoretical and simulation results for the series matching capacitor $C_{m,in}$ are plotted in Fig. 4.

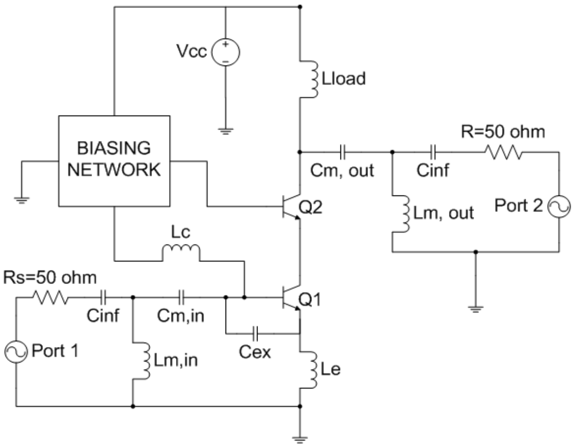

Figure 2. The adopted LNA topology to validate the theoretical expectations about the tunability introduced with the series matching capacitor

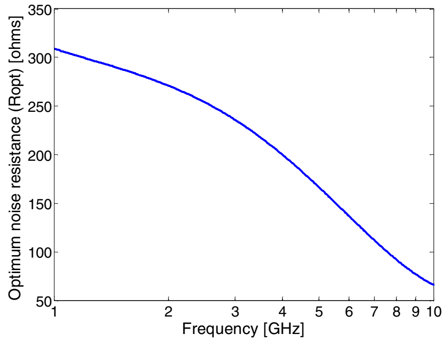

Figure 3. Optimum noise resistance R_{OPT} vs. frequency

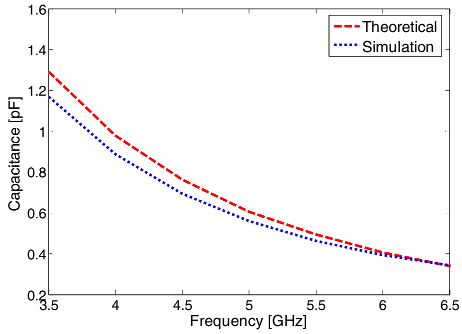

Figure 4. The comparison of the theoretical and simulation based results for the input series matching capacitor $C_{m,in}$

It is easy to observe that the theoretical projection closely supports the design outcomes. The range of frequencies from 3.5 GHz to 6.5 GHz has been dictated by S_{11} which goes beyond -10 dB for frequencies outside of this range. $C_{m,in}$ requires to be tunable between 350 fF to 1.3 pF, whereas the associated $C_{m,out}$ value should vary between 400 fF to 1.6 pF.

III. CHARACTERIZATION OF THE TUNABLE CAPACITORS BY MEANS OF THEIR QUALITY FACTORS

After the concept of tunability has been demonstrated the next step should be determining the specifications on the tunable capacitor imposed by the design constraints. Physically, building variable reactive elements has been a challenge because maintaining a high performance over a certain frequency range thereby providing the necessary capacitance is not trivial. In this paper, rather than discussing the fabrication problems of the tunable components, the specifications derived from the design requirements will be described that can provide an enlightening viewpoint to the complex production process.

The most important characteristic of a reactive component is its quality factor (Q-factor). It basically shows the amount of inherent resistive loss incorporated to the capacitive/ inductive element. For a capacitor, the Q-factor can be defined as in (14):

$$Q=\frac{1}{\omega R_s C_s} \tag{14}$$

High Q-factors indicate less noise contribution to the integrated system by the tunable element. Nevertheless, such elements incur a great cost to the budget, making the tunability option unfeasible. It is crucial to determine the minimum permissible Q-factor for the tunable component that will result in an optimum combination of specifications. For this purpose, the topology shown in Fig. 2 has been redesigned using the developed design procedure, such that all components except the matching capacitor $C_{m,in}$ are taken from the IBM 7HP design kit. $C_{m,in}$ is modeled as an ideal capacitor in series with a resistor, which will determine its Q-factor. Since the effect of $C_{m,out}$ to the performance is minor, it is modeled as a design kit capacitor. By sweeping the resistance of $C_{m,in}$, it is possible to get the plots of design parameters such as NF, the gain (S_{21}) and S_{11} vs. the Q-factor. The simulations are done at three different frequencies, 3.5 GHz, 5 GHz, and 6.5 GHz for which the resulting graphs are shown in Fig. 5, Fig. 6, and Fig. 7.

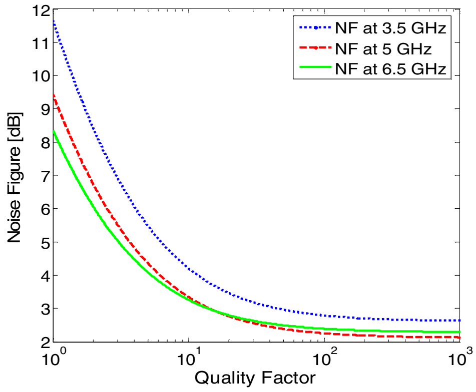

Figure 5. The noise figure performance of the LNA vs. the quality factor of the series matching capacitor $C_{m,in}$ at 3.5 GHz, 5 GHz and 6.5 GHz.

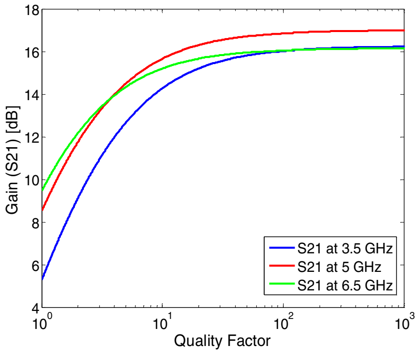

Figure 6. The gain (S_{21}) performance of the LNA vs. the quality factor of the series matching capacitor $C_{m,in}$ at 3.5 GHz, 5 GHz and 6.5 GHz.

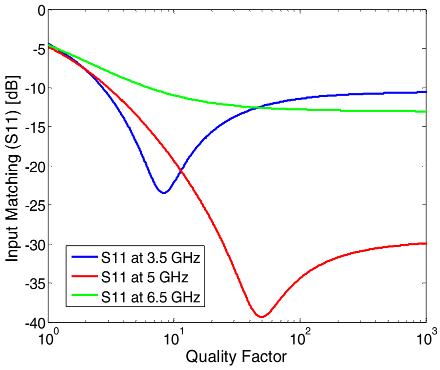

Figure 7. The input matching (S_{11}) performance of the LNA vs. the quality factor of the series matching capacitor $C_{m,in}$ at 3.5 GHz, 5 GHz and 6.5 GHz.

To begin with, the NF graph (Fig. 5) will be interpreted. The behavior of all curves can be described in three regions. The first region for which the Q-factor is higher than 100, NF curves are almost constant indicating that the nonlinear contribution of the tunable component is at the minimum level. In the second region, where the Q-factor changes between 10 and 100, NF values experience a minor increase (around 1 dB at Q-factor is equal to 10). Finally, in the last region where Q-factor is less than 10 huge increases in NF results of the LNA are to observe. Based on this analysis, it can be concluded that a minimum Q-factor of 10 should be fulfilled by the variable capacitor throughout the tunable band. Practically, a Q-factor of 10 for a capacitor is relatively straightforward to achieve at narrowband, however, the main challenge here would be to keep it at this constant value across the whole frequency range.

Similar to the NF behavior of the matching network, the gain response -presented in Fig. 6- deteriorates when the Q-factor of the tunable capacitor drops below 10. If the Q-factor is above this threshold, for all three frequencies, the gain ranges between 14-17 dB.

The S_{11} response shown in Fig. 7 is more complex compared to the other quantities. At lower frequencies, an optimum Q-factor, Q_{OPT} that results in the best S_{11} value can be observed. For 5 GHz, Q_{OPT} is about 50 whereas for 3.5 GHz, it becomes 10. Thus, as the frequency gets scaled down below 3.5 GHz, introducing more resistive loss with the capacitor can be beneficial to keep S_{11} below -10 dB. However, the tradeoff would be a significantly increased NF and NF_{min} as well as a reduced gain. Also, Fig. 6 reveals that at higher frequency, Q_{OPT} ceases to exist and S_{11} behavior becomes similar to the NF response, mainly because any additional parasitic resistance augments the difference between the input resistance R_{IN} and the source resistance R_S.

IV. CONCLUSION

The input matching network is very crucial for the design of LNAs. Alternative matching structures can be useful to realize multiband operation. In this case, a π-matching network with a series capacitor is shown to be promising both through a theoretical and design based approach in terms of noise figure and input return ratio of tunable LNA applications. The Q-factor characterization indicated that a variable capacitor could satisfy the design specifications over a 3 GHz band with the center located at 5 GHz, if it exhibits a moderate Q-factor (Q> 10) for the entire tunability range. If the center frequency would be selected higher, the whole C band (4 GHz to 8 GHz) could be spanned through a single LNA circuit.

REFERENCES

[1] S. P. Voinigescu, M. C. Maliepaard, J. L. Showell, G. E. Babcock, D. Marchesan, M. Schroter, P. Schvan. D. L. Harame, “A Scalable High-Frequency Noise Model for Bipolar Transistors with Application to Optimal Transistor Sizing for Low-Noise Design”, *IEEE J. Solid-State Circuits*, Vol. 32, No. 9, pp. 1430-1439, 1997.

[2] D. K. Schaeffer and T. H. Lee, “A 1.5 V, 1.5 GHz CMOS Low Noise Amplifier”, *IEEE J. Solid-State Circuits*, Vol. 32, No. 5, pp. 745-759, 1997.

[3] T. K. Nguyen, C. H. Kim, M. S. Yang, and S. G. Lee, “CMOS Low-Noise Amplifier Design Optimization Techniques”, *IEEE Trans. Microw. Theory Tech.*, Vol. 52, No. 5, pp. 1433-1442, 2004.

[4] G. Vandersteen, L. Bos. P. Dobrovolny, “Scaling Friendly Design Methodology for Inductively Degenerated RF Low-Noise Amplifiers”, in *Proc. 2007 2nd European Microwave Integrated Circuits Conf.*, Munich, Germany, pp. 223-226

[5] H. Hashemi and A. Hajimiri, “Concurrent Multiband Low-Noise Amplifiers- Theory, Design, and Applications”, *IEEE Trans. Microwave Theory and Techniques,* Vol. 50, No.1, pp. 288-301, 2002.

[6] N. Ahsan, A. Ouacha, J. Dabrowski, C. Samuelsson, “Dual Band Tunable LNA for Flexible RF Front End”, in *Proc. 2007 5th of Intl. Bhurban Conference on Applied Sciences & Technology*, pp. 19-22.

[7] J. Danson, C. Plett, N. Tait, “Using MEMS Capacitive Switches in Tunable RF Amplifiers”, *EURASIP J. Wireless Communications and Networking,* Vol. 2006, pp.1-9, 2006.

[8] S. H. Shin and H. J. Yoo, “A Multistandard RF Front-End Using Varactor Controlled Tunable Interstage Matching Network”, in *Proc. 2007 IEEE Radio and Wireless Symposium*, pp. 181-184.

[9] M. B. Yelten, “Theoretical Analysis and Design Methodologies for Low Noise Amplifiers based on Tunable Matching Networks”, M.S. Thesis, NC State Univ. Dept. of Elect. & Comput. Eng., Raleigh, 2008.

[10] M. B. Yelten and K. G. Gard “A Novel Design Procedure for Tunable Low Noise Amplifiers”, in *Proc. 2009 10th Annual IEEE Wireless and Microwave Conf., Clearwater, Florida, USA,* in press.