Accurate Estimation of Digital Communication System Metrics — SNR, EVM and ρ in a Nonlinear Amplifier Environment

Khaled M. Gharaibeh, Kevin G. Gard and Michael B. Steer
Dept. of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC 27695
Tel. 919-515-5191, FAX 919-515-2285, e-mail: {kmgharai,kggard,mbs}@ncsu.edu

Abstract - **A nonlinear spectral analysis technique that enables digital communication system metrics; SNR, EVM and the waveform quality factor (ρ) to be related to in-band distortion spectrum is presented. System metrics are estimated from the measured output power and in-band distortion power. The estimated metrics are verified by direct measurements of each metric using a Vector Signal Analyzer (VSA) performed on a forward-link IS-95 signal. Estimated system metrics are in excellent agreement with measured values.**

I. Introduction

Predicting system performance metrics such as Signal-to-Noise Ratio (SNR), Error Vector Magnitude (EVM) and the waveform quality factor (ρ) is usually a difficult task when the signal is processed by a nonlinear amplifier. A high dynamic range receiver and demodulator are required to detect the baseband symbols necessary for calculating waveform quality metrics. It is therefore desirable to accurately estimate these metrics from measurements which do not require a sophisticated digital receiver.

The nonlinear behavior of RF and microwave amplifiers result in in-band (or co-channel) distortion which is manifested as a degradation of SNR and ultimately as a degradation of Bit Error Rate (BER). ρ and EVM are measures of the fidelity of a digital communication system and are related to in-band distortion and SNR. Characterizing in-band distortion and its relation to the system metrics require the correlated and the uncorrelated components of the output spectrum to be identified. The correlated output component consists of an amplified version of the input waveform with gain compression/expansion and represents the useful part of the output that leads to correct detection of the received data. On the other hand the uncorrelated part adds to the system interference in a similar way to that of Additive White Gaussian Noise (AWGN). Thus the correlated and the uncorrelated nonlinear output components contribute differently to the degradation of system SNR, EVM, ρ and ultimately system BER.

In [1] we reported a behavioral modeling technique that enables the effective in-band distortion to be predicted from basic amplifier characteristics without any assumption on signal statistics such as the Gaussian assumption. The technique is based on separating the correlated and uncorrelated components of the output spectrum and hence it allows the in-band distortion to be identified. In this paper we utilize this technique and develop a formulation of in-band distortion and its relation to the most common metrics of system performance; SNR, EVM and ρ. In addition we develop the relationship among the three metrics. These relationships are verified by direct measurements performed on an IS-95 system using a vector signal analyzer and vector signal generator. Measurement of in-band distortion of a high frequency signal is based on a feed-forward cancellation measurement scheme to remove the correlated component of the output spectrum.

II. Relationship Between System Metrics and In-band Distortion

In order to develop the relationship between the co-channel distortion and system metrics we need to write the output autocorrelation function (which is directly related to the output spectrum) as the sum of uncorrelated components. Using the analysis in [1] the output of a nonlinear amplifier is expressed as a useful component $\tilde{y}_c(t)$ correlated with the input signal, an uncorrelated distortion component $\tilde{y}_d(t)$:

$$\tilde{y}(t) = \tilde{y}_c(t) + \tilde{y}_d(t). \tag{1}$$

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The useful part of the signal $\tilde{y}_c(t)$ consists of the linearly amplified version of the input signal and the correlated part of the spectral regrowth term. The correlated portion of the distortion does not contribute to distortion noise but rather affects the signal level in a manner akin to gain saturation or enhancement of discrete tones. The uncorrelated part of the output $\tilde{y}_d$ is additive distortion noise and affects system performance in a similar way to AWGN. Thus both the correlated and uncorrelated parts of the output affects the output SNR and BER in different ways. Now, using the orthogonal behavioral model, the output autocorrelation function can now be written as

$$R_{\tilde{y}\tilde{y}}(\tau) = R_{\tilde{y}_c\tilde{y}_c}(\tau) + R_{\tilde{y}_d\tilde{y}_d}(\tau) \quad (2)$$

where

$$R_{\tilde{y}_c\tilde{y}_c}(\tau) = |c_1|^2 R_{u_1 u_1}(\tau)$$

and

$$R_{\tilde{y}_d\tilde{y}_d}(\tau) = \sum_{n=3}^{N} |c_n|^2 R_{\tilde{u_n}\tilde{u_n}}(\tau).$$

The output Power Spectral Density (PSD) is obtained from the Fourier transform of 2:

$$S_{\tilde{y}\tilde{y}}(f) = S_{\tilde{y}_c\tilde{y}_c}(f) + S_{\tilde{y}_d\tilde{y}_d}(f). \quad (3)$$

The output spectrum is therefore the sum of the spectra of the uncorrelated signal components of the output waveform.

At the receiver, the received signal consists of the amplified transmitted signal and an AWGN component. Therefore, the signal at the receiver can be written in complex envelop form as

$$\tilde{r}(t) = \tilde{y}_c(t) + \tilde{y}_d(t) + \tilde{n}(t) \quad (4)$$

where $\tilde{n}(t)$ is AWGN component with power spectral density of N_o. Note that the three output components are now uncorrelated and therefore the uncorrelated output distortion is treated as an additive noise similar to AWGN. The total system noise which affects the output signal quality (and hence system BER) consists of uncorrelated in-band distortion and AWGN as will be seen in the following. In the following we derive the most common measures of in-band distortion using the above analysis.

A. Signal-to-Noise Ratio (SNR)

The effective system SNR is defined as the ratio of signal power to total noise power including the power of the in-band distortion terms. It can be expressed in terms of the PSD's of the uncorrelated output components of (4) as

$$\mathrm{SNR} = \frac{\int_{-B/2}^{B/2} S_{\tilde{y}_c\tilde{y}_c}(f)df}{\int_{-B/2}^{B/2} S_{\tilde{y}_d\tilde{y}_d}(f)df + N_0 B} \quad (5)$$

where $S_{xx}(f)$ is the PSD of signal $x(t)$ and B is the bandwidth of the input signal. Note that SNR is a function of both the nonlinear distortion and the energy per bit-to-AWGN ratio E_b/N_0. Evaluating the effective SNR is important to determine the system BER and the system Noise Figure (NF). These parameters are usually estimated assuming a linear AWGN channel however nonlinear distortion increases the system BER for a fixed AWGN power.

B. CDMA Waveform Quality Factor (ρ)

The waveform quality factor is a measure of the correlation between a scaled version of the input and the total in-channel output waveforms. Therefore, using the above formulation, the waveform quality factor is defined as:

$$\begin{aligned} \rho &= \frac{E[\tilde{y}(t)\tilde{y}_c^*(t)]^2}{E[|\tilde{y}(t)|^2]E[|\tilde{y}_c(t)|^2]} \\ &= \frac{\int_{-B/2}^{B/2} S_{\tilde{y}_c\tilde{y}_c}(f)df}{\int_{-B/2}^{B/2} S_{\tilde{y}_c\tilde{y}_c}(f)df + \int_{-B/2}^{B/2} S_{\tilde{y}_d\tilde{y}_d}(f)df + N_0 B} \end{aligned} \quad (6)$$

where the in-band portion of the signals $y(t)$, $y_c(t)$ and $y_d(t)$ is used in this expression. Note that ρ can be directly related to the effective SNR and it measure the fraction of the useful part of the signal at the receiver. Comparing (5) and (6), the relationship between ρ and SNR can be written as

$$\rho = \frac{\mathrm{SNR}}{\mathrm{SNR}+1}.$$

In an IS-95 system ρ is usually measured when only the pilot channel is transmitted [2]. In this case, the NBGN assumption of the signal model is not valid and therefore the estimated ρ using the properties of the Gaussian higher order moments does not lead to an accurate estimation of ρ which gives the above analysis its importance.

C. Error Vector Magnitude (EVM)

Error Vector Magnitude (EVM) is a common figure of merit for system linearity in digital wireless communication standards (including GSM, NADC, IS-95 and WCDMA systems) where a maximum level of EVM is specified. EVM is defined in the context of

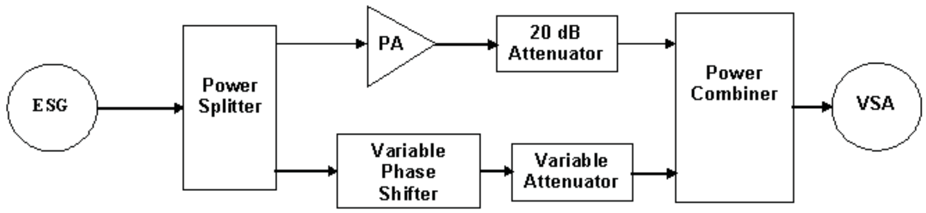


Fig. 1. Feed-forward cancellation measurement setup.

digitally modulated signals where it is a measure of the departure of signals constellation from its ideal reference because of nonlinearity. Nonlinearity results in compression/expansion and rotation of the signal constellation. EVM can be defined using (4) in terms of the signal and noise power as

$$\begin{aligned} \mathrm{EVM} &= \sqrt{\frac{E[\tilde{y}_d^2(t)] + E[\tilde{n}(t)^2]}{E[y_c^2(t)]}} \\ &= \sqrt{\frac{\int_{-B/2}^{B/2} S_{\tilde{y}_d\tilde{y}_d}(f)df + N_0 B}{\int_{-B/2}^{B/2} S_{\tilde{y}_c\tilde{y}_c}(f)df}}. \qquad (7) \end{aligned}$$

Using (5) and (6) EVM can be related to SNR and ρ as follows

$$\mathrm{EVM} = \sqrt{\frac{1}{\mathrm{SNR}}}$$

and

$$\mathrm{EVM} = \sqrt{\frac{1}{\rho} - 1}.$$

Note that EVM and ρ are directly related to SNR and hence the ability of the receiver to perform reliable detection of the transmitted data.

III. MEASUREMENTS

The measurements presented here were performed using Agilent E4438C vector signal generator, Agilent E4445A PSA spectrum analyzer and 89600S vector signal analyzer. The in-band distortion is measured using a feed-forward cancellation setup as shown in Fig. 1. The input signal is a forward link IS-95 signal with the pilot only is generated using Agilent ESG 4438C vector signal generator. The signal is split using a power splitter into two branches. The first is amplified by the nonlinear PA under test and the second is used after phase reversal to cancel the linear component of the output. The variable phase shifter and the variable attenuator are used to adjust the linear input level and phase to cancel the linear output of the amplifier. The resulting output at the power combiner consists of the total uncorrelated distortion (in-band and out-of-band). The in-band distortion is measured as the uncorrelated distortion within the signal bandwidth using an Agilent E4445A PSA spectrum analyzer. Fig. 2 shows measured total output and the uncorrelated distortion spectra. The measured in-band distortion as a function of the output power is shown in Fig. 3(a). The minimum measured in-band distortion (at low power levels) is limited by the finite cancellation that the feed-forward approach provides [3]. This is due to the finite gain and phase matching that can be achieved by the instrument and the measurement setup.

The SNR, ρ and EVM were estimated from the in-band distortion and verified by directly using a VSA. A measured SNR of 32 dB at small signal (which represents a lower limit of the noise power in the VSA measurements) was included in the estimate of the metrics. Figs 4 (b), (c) and (d) show a a good agreement between measured and estimated values.

IV. CONCLUSION

A technique to directly relate system metrics to nonlinear distortion has been developed and verified. SNR, EVM and ρ were estimated from in-band distortion measured using feed-forward cancellation. The estimated metrics are in excellent agreement with direct measurements of system metrics using a VSA. The simplicity of the measurement enables the metrics to be predicted using simple lab equipment.

REFERENCES

[1] K. M. Gharaibeh and M. B. Steer, "Performance Analysis of Direct Sequence Spread Spectrum (DS-SS) CDMA System With Amplifier Nonlinearities." *IEEE Radio and Wireless Conference, RAWCON* 2004, Sep. 2004.

[2] V.Aparin, "Analysis of CDMA signal spectral regrowth and waveform quality," *IEEE Trans. Microwave Theory Techn.*, Vol. 49, pp. 2306–2314, Dec. 2001.

[3] P. B. Kenington, High Linearity Amplifier Design, Massachssets: Artech House Inc., 2000.

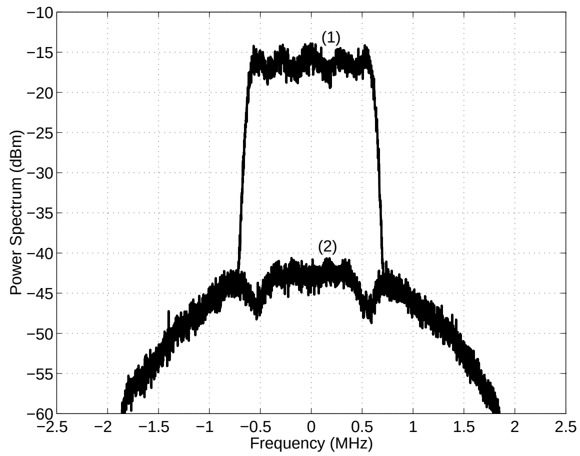


Fig. 2. (a) Measured spectrum: (1) total output spectrum and (2) in-band distortion (output of the feed-forward loop).

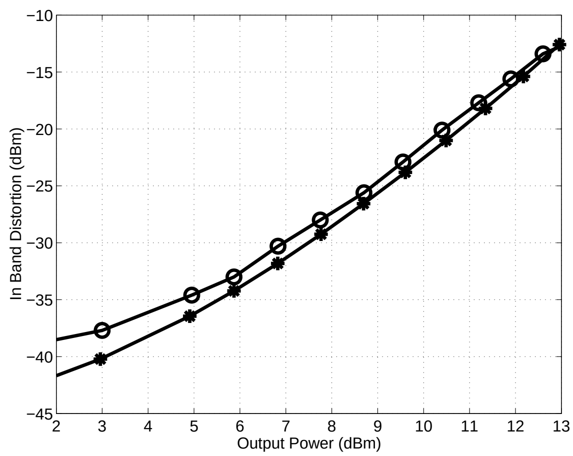


Fig. 3. In-band distortion power; ○ : measured; and ∗: simulated.

[4] K. Gard, H. Gutierrez, M. B. Steer, "Characterization of spectral regrowth in microwave amplifiers based on the nonlinear transformation of a complex Gaussian process," *IEEE Trans. Microwave Theory Techn.*, Vol. 47, pp. 1059–1069, July 1999.

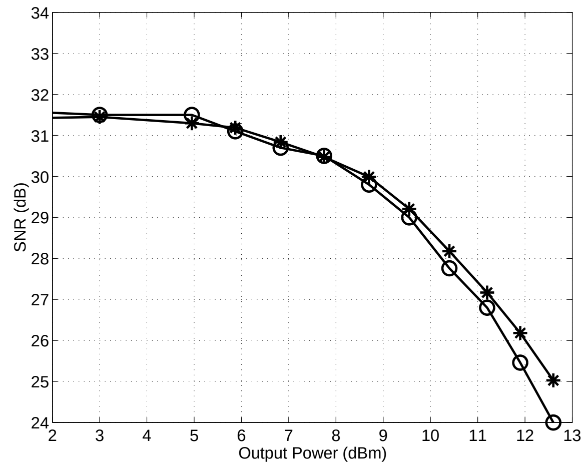
[5] N. Blachman, "The uncorrelated output components of a nonlinearity *IEEE Trans. Inf. Theory*, Vol. 14, pp. 250–255, March 1968.

[6] P.M. Lavrador, N.B. de Carvalho and J.C.Pedro, "Evaluation of signal-to-noise and distortion ratio degradation in nonlinear systems," *IEEE Trans. Microwave Theory Techn.*, Vol. 49, pp. 813–822, March 2004.

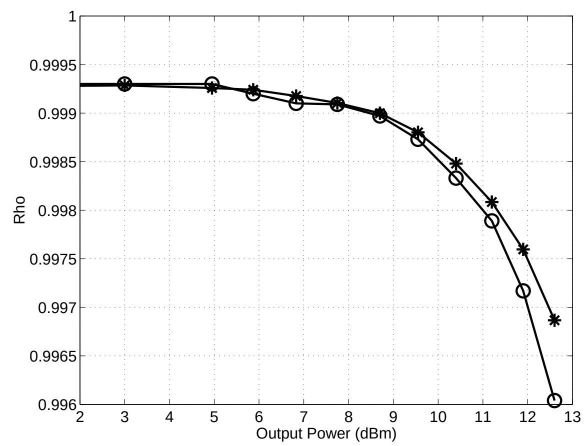
[7] J. S. Bendat, *Nonlinear System Analysis and Identification*, John Wiley and Sons, NY 1990.

[8] T. S. Rappaport, Wireless Communications: Principles and Practice, New Jersy: Prentice Hall Inc., 1996.

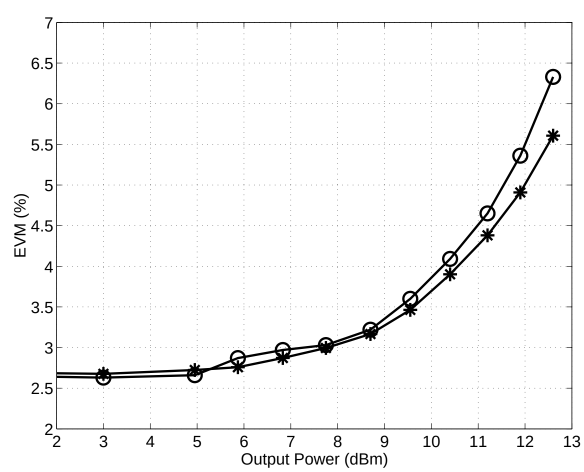
[9] M. Jeruchim, P. Balaban and S. Shanmugan, Simulation of Communication Systems, New York: Kluwer Academic/Plenum Publishers, 2000.



(a)



(b)



(c)

Fig. 4. System metrics: (a) SNR, (b) ρ and (c) EVM vs. output power; ○ : measured; and ∗: estimated using measured in-band distortion.