Simple, Broadband Relative Phase Measurement of Intermodulation Products

Aaron Walker, Michael Steer, and Kevin Gard
Department of Electrical and Computer Engineering
North Carolina State University
Raleigh, NC 27695-7914 USA
email: alwalke3@ncsu.edu mbs@ncsu.edu kggard@ncsu.edu

Abstract— **Single tone and two-tone tests are used to characterize the nonlinear response of RF and microwave systems. One-tone testing is relatively straightforward. In contrast, two-tone testing results in the generation of intermodulation frequencies and generally only the magnitudes of these are measured. However full characterization requires that phase be extracted as well as magnitude. Measurement of the phase of intermodulation frequencies has historically been difficult. In this paper we present a broadband technique for performing the relative phase measurement of the intermodulation frequency components generated in a two-tone test. The method uses three phase-locked sources and common equipment found in a microwave laboratory.**

I. Introduction

In the past, measurement of the phase of frequency components arising from large-signal excitation of RF and microwave devices has been difficult to perform. Since nonlinear behavior results in frequency components at the device output not present in the stimulus, the biggest problem with measuring phase is that there is no reference signal in the stimulus to use as a phase reference for the new frequency content. Unlike small-signal measurements performed with a vector network analyzer, no convenient source of the new frequency content exists as a reference. Omitting the phase of the additional frequency content limits the accuracy of system models. Measurement of the phase of all output spectral products of interest increases the level of knowledge about the DUT allowing designers to better understand the operation of the DUT in the large-signal regime.

There are two main attributes of the measurement system presented here. First, the system quantifies phase and amplitude information on the frequency content produced by nonlinear behavior, allowing development of models that can more closely track the operation of real devices and systems. Second, the method is simple in both setup and operation as well as requiring only a small number of standard benchtop instruments already available in RF and microwave laboratories. This attribute is in contrast to existing instruments and measurement setups that measure the phase of intermodulation products. These methods either introduce complexity through reference devices, or circuit schemes to produce the phase reference signal, or require expensive instrumentation in order to perform the measurement.

II. Intermodulation Phase Measurement Background

Though none are widely used, several methods have been developed to measure the relative phase of nonlinear products generated during large-signal excitation. The majority of these techniques can only measure the phase of harmonics of the fundamental signal and thus cannot be used to investigate inband nonlinear behavior. The most mature of these methods is based on the Nonlinear Vector Network Analyzer, NVNA, introduced in 1994, [1]. This system digitizes downconverted frequency bands around the fundamental and its harmonics with an 8 MHz bandwidth. Until recently, this system was only used to measure the phase of the harmonics produced by a nonlinear DUT, but was used in [2] to measure the phase of intermodulation products in two-tone excitation experiments. The primary limitation of using this instrument is its high cost, much more than that of a traditional network analyzer.

The other measurement systems published in literature have not undergone the extensive development of the NVNA. The setups used in [3] and [4] can only be used for wideband characterization of devices since these only measure the phase of harmonic products. The system in [3] uses a straightforward measurement of the phase using a network analyzer with reference signals provided by a multi-harmonic signal generator. The system in [4] is much more complicated and uses frequency multipliers to supply the reference signals where phase is measured by a Microwave Transition Analyzer (MTA). The methods in [5] – [9] are designed to measure the phase of the intermodulation products directly. Those in [5] and [6] use reference nonlinearities as the source of the phase reference while those in [7], [8], and [9] use multiple mixers to generate the reference signals. The system in [8] uses five signal generators to produce the necessary reference signals and an accompanying large amount of interconnect and signal routing components to complete the measurement apparatus.

Reviewing these measurement setups reveals several important issues. First, the simplest of these methods rely on reference nonlinear devices to provide the reference signals, which implies that these devices have been characterized to a level sufficient for instrument grade calibration, an assumption that may be difficult in practice. Second, the more complicated schemes require extensive calibration due to their high number of components in the system or the use of multiple frequency mixers. The use of mixers in general has considerable implications as these devices themselves are highly nonlinear and thus care must be taken such that they do not introduce nonlinear behavior similar to that produced by the DUT, [9]. Most importantly, many of the systems require a large amount of instrumentation to implement, with many requiring a vector network analyzer (VNA), several employing multiple spectrum analyzers and sources, and others requiring MTAs. While these instruments are common in most microwave laboratories, utilization of so many instruments in complicated setups represents a high cost in both capital and time of use.

The goal of the measurement system presented here is to provide a simple, broadband measurement apparatus to determine the relative phase of the intermodulation products, specifically the IM3 products. We chose to avoid the use of reference nonlinearities in our system since the characterization of the reference introduces additional sources of error and the complexity of keeping the power level of the stimulus to the reference device constant during swept power measurements. We also chose to employ the minimum equipment necessary to perform the measurement which cuts down the implementation cost and calibration complexity. The method presented here uses similar concepts as the reference nonlinearity methods. The main difference in our method lies in the use of a signal source with control over amplitude and carrier phase to provide the cancellation signal. The relative phase is determined by cancelling spectral products from the DUT through adjustment of a reference signal phase and amplitude combined with the DUT output.

III. IM3 Phase Measurement and Calibration

The phase measurement method described in the previous section is performed with the system shown in Fig. 1. The system consists of three identical, phase-locked RF sources, (Marconi 2024), hybrid combiners, isolators, and a spectrum analyzer. The RF sources provide both the stimulus to the DUT and the mechanism employed to measure the phase of any of the discrete tones in the DUT output. Measurement of phase is performed by cancelling the product of interest at the DUT output with Source 3 by varying its amplitude and phase with internal elements. The three sources share a common 10 MHz reference frequency with the spectrum analyzer. The common reference allows the synthesizers in each instrument to stay locked in frequency to other instruments at the same frequency, however this does not imply that the instruments are phase locked when frequencies are changed. In our system design, several synthesizers were used to provide the stimulus and cancellation tones. It was observed that when using different models of synthesizers, considerable phase drift occurred between the sources such that the phase measurement provided by cancellation was not repeatable. Thus, while the sources remained locked to the same frequency, having identical average time-varying phase, the instantaneous phase of the synthesizers did not track each other and thus led to variations in relative phase apparent in less than one minute of time. By using three identical sources with the same frequency reference, the phase drift issue was eliminated and the stability of phase between sources could be measured in hours.

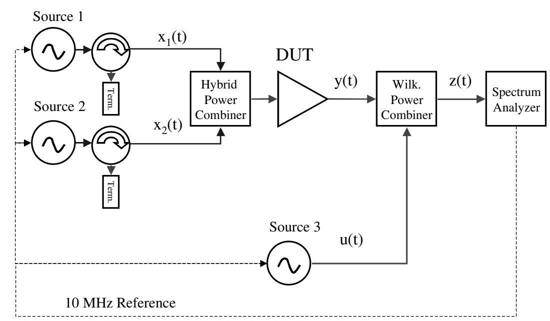

Fig. 1. Measurement system block diagram incorporating phase-locked signal sources.

Calibration of the measurement setup requires removing the effect of the sources' phase shift with changes in output power and choosing a suitable reference for the relative phase measurement. The former requires characterization of the source output phase as a function of power and frequency, while the latter is based on the model for the phase of IM3 products at small-signal. In [5], the authors showed that the relative phase of the IM3 products was constant for the weakly nonlinear region of the input stimulus. This agrees with the behavior observed during single tone AM-AM AM-PM analysis that shows nonlinear behavior is largely resistive, i.e. without phase shifting, for small-signal inputs. Thus, for the measurement procedure here, we take the reference phase as that at a designated small-signal input level.

A. *Source Operation*

The sources introduce phase shifts in the supplied RF stimulus and cancellation tones through two different mechanisms. The first results from switches in fixed attenuator settings in the source output path to accommodate the wide range in power settings available. Correction for this effect is straightforward as the attenuator switch points occur at deterministic output power levels. In addition, the attenuator lock feature of the sources removes this effect for the generators that supply the DUT stimulus as the power control range of this feature allows a large sweep in source output power ($\sim$ 30 dB). Attenuator lock cannot be used on the source providing the IM3 cancellation tone as the range in power required for this source is much greater than 30 dB. The second mechanism for phase insertion by the sources arises from a linear shift in output phase with power change (power in dB).

Both of the phase shift responses were measured using a network analyzer with an S-parameter test set, where one of the sources was used to supply the reference signal for the test set and the other source drove port 2 of the test set. In this setup, shown in Fig. 2, the relative phase between the reference source and the source at port 2 is measured as the phase of S_{21}. Figure 3 displays the measured normalized phase, (actually delay), for a single frequency across a power range of -70 to $+13$ dBm. As shown, six distinct attenuator switch points occur over this power range. After removing the phase attributed to the attenuator steps, the linear nature of the phase shift with power is clearly evident. Figure 4 reveals that this effect is essentially constant across the frequency band with the phase shift normalized by the frequency of operation. The normalized phase shift with power shows that this response is fundamental to the design of the sources used as it is nearly constant across the frequency band $400-2400$ MHz.

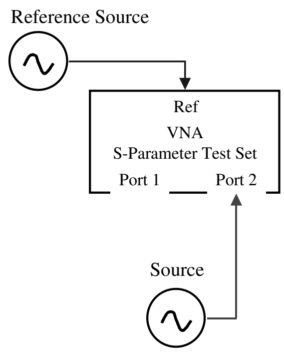

Fig. 2. Test setup for measuring phase calibration data for RF sources.

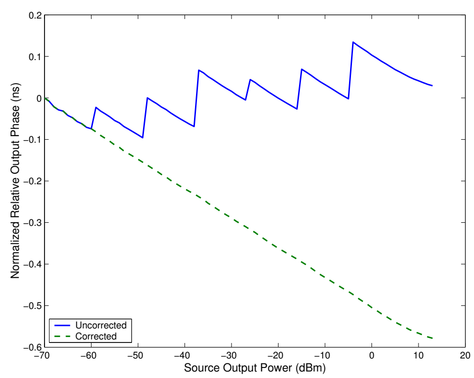

Fig. 3. Measured normalized phase insertion of RF source and step attenuator corrected phase insertion at 400 MHz.

B. Effect of Source Phase Shift on Measurements

The phase of the source output, ignoring the attenuator steps, can be expressed as

$$\phi_i(P) = m_s(\omega) \cdot P_{dB} + \phi_c \ , \tag{1}$$

where m_s is the linear phase shift with power and a weak dependence on output frequency ω and ϕ_c is a constant phase associated with the initial phase of the source when it locks to the desired frequency. In general, this constant phase can be taken as being zero. For the measurement system presented, the phase at some small-signal reference power is set to zero

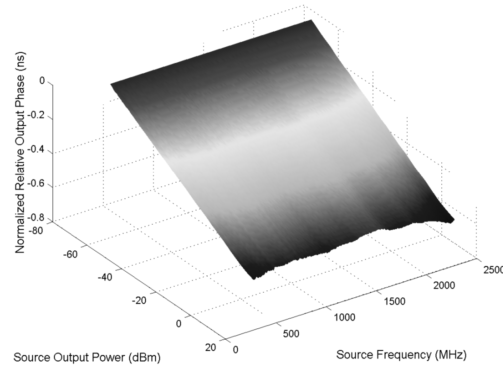

Fig. 4. Corrected phase insertion of RF source vs. carrier frequency and output power demonstrating linear phase shift with output power and weak dependence on carrier frequency.

such that $\phi_i(P_{ref}) = 0$. Now the expression for the output of the sources becomes

$$x_i(t) = A_i \cos(\omega_i t + \phi_i(P)) \ , \tag{2}$$

Now putting the source output into complex exponential form

$$x_i(t) = \frac{1}{2} \sum_{\substack{q=-1 \\ q \neq 0}}^{1} A_q(P) e^{j(\omega_q t + \phi_q(P))} \ , \tag{3}$$

with $\omega_{-q} = -\omega_q$, $\phi_q(P) = m_s(\omega) \cdot P_{dB} + \phi_c$, and $\phi_{-q} = -\phi_q$.

With the source output in the form of (3), we can use the standard analysis of a nonlinearity represented as a power series with complex coefficients to determine the effect of the source phase shift on the DUT output. For an arbitrary number of stimulus tones Q, the DUT output for a particular polynomial order is given as

$$y_n(t) = a_n \left[\frac{1}{2} \sum_{\substack{q=-Q \\ q \neq 0}}^{Q} A_q(P) e^{j(\Phi_q)} \right]^n \ , \tag{4}$$

where $\Phi_q = \omega_q t + m_s(\omega_q) \cdot P_{dB} + \phi_c$. Now the response for the n-th order is

$$y_n(t) = \frac{a_n}{2^n} \sum_{q_1=-Q}^{Q} \cdots \sum_{q_n=-Q}^{Q} A_{q_1} \cdots A_{q_n} e^{j(\Phi_{q_1}(P) + \ldots + \Phi_{q_n}(P))} \ , \tag{5}$$

with the total response given by

$$y(t) = \sum_{n=0}^{N} y_n(t) \ . \tag{6}$$

Since the phase that is measured is a relative phase to that at a particular power, the expression for Φ_q becomes,

$$\Phi_q = \omega_q t + m_s(\omega_q) \cdot \Delta P_{dB} \tag{7}$$

Now expanding the first terms of (6), taking into account the effect of the complex polynomial coefficient we have

$$
\begin{aligned}
&y(t) = \\
&\frac{|\,a_3\,|}{2^3} \sum_{q_1=-Q}^{Q} \sum_{q_2=-Q}^{Q} \sum_{q_3=-Q}^{Q} A_{q_1} A_{q_2} A_{q_3} e^{(\Phi_{q_1}+\Phi_{q_2}+\Phi_{q_3} \pm \phi_{a_3})} \\
&+ \frac{|\,a_5\,|}{2^5} \sum_{q_1=-Q}^{Q} \cdots \sum_{q_5=-Q}^{Q} A_{q_1} \cdots A_{q_5} e^{(\Phi_{q_1}+\cdots+\Phi_{q_5} \pm \phi_{a_5})} \\
&+ \ldots \;, \qquad (8)
\end{aligned}
$$

where $\pm\phi_n$ signifies that the phase of the complex coefficient contributes a complex conjugate term to the phase of positive and negative frequencies. The terms that contribute to the intermodulation product at $2\omega_1 - \omega_2$ for a two-tone stimulus have a form such that the frequency component of $\Phi_{q_1} + \cdots + \Phi_{q_n} = 2\omega_1 - \omega_2 + \phi$, where ϕ is a phase term dependent on the term under consideration. For a third-order contributor, there is only one set of q_i values that will produce this frequency (although there are multiple permutations of this set within the sum in (8)), i.e. $q_1 = 1,\ q_2 = 1,\ q_3 = -2$. For this set of mixing products, the phase component is

$$
\begin{aligned}
\Phi_{q_1} + \Phi_{q_2} + \Phi_{q_3} &= \omega_1 t + m_s(\omega_1) \cdot \Delta P_{1,dB} \\
&+ \omega_1 t + m_s(\omega_1) \cdot \Delta P_{1,dB} \\
&- \omega_2 t - m_s(\omega_2) \cdot \Delta P_{2,dB} \\
\\
\Phi_{q_1} + \Phi_{q_2} + \Phi_{q_3} &= (2\omega_1 - \omega_2)t \\
&+ \Delta P_{dB}\ (2m_s(\omega_1) - m_s(\omega_2)) \\
\\
\Phi_{q_1} + \Phi_{q_2} + \Phi_{q_3} &= (2\omega_1 - \omega_2)t + \Delta P_{dB} \cdot m_s(\omega_1) \;,
\end{aligned}
\qquad (9)
$$

with the last step allowed since $m_s(\omega_1) \cong m_s(\omega_2)$. Similarly for one of the fifth-order contributors, $q_1 = 1,\ q_2 = 1,\ q_3 = 2,\ q_4 = -2,\ q_5 = -2$, the phase is

$$
\Phi_{q_1} + \cdots + \Phi_{q_5} = (2\omega_1 - \omega_2)t + \Delta P_{dB} \cdot m_s(\omega_1) \;. \qquad (10)
$$

This demonstrates that for all of the odd-ordered contributors, the effect of the linear phase shift with power is the same and adds a single constant term to each of the phase components. Expressing the individual odd-ordered contributors in phasor form one obtains

$$
\begin{aligned}
Y_{\mathrm{IM3L}}(2\omega_1 - \omega_2) &= \frac{|\,a_3\,|}{2^{3-1}} c_{3,\mathrm{IM3L}}\ A^3 e^{j(\Delta P_{dB} \cdot m_s(\omega_1) + \phi_{a_3})} \\
&+ \frac{|\,a_5\,|}{2^{5-1}} c_{5,\mathrm{IM3L}}\ A^5 e^{j(\Delta P_{dB} \cdot m_s(\omega_1) + \phi_{a_5})} \\
&+ \ldots \;, \qquad (11)
\end{aligned}
$$

or

$$
\begin{aligned}
Y_{\mathrm{IM3L}}(2\omega_1 - \omega_2) = e^{j(\Delta P_{dB} \cdot m_s(\omega_1))} \cdot \\
\Big[\frac{|\,a_3\,|}{2^{3-1}} c_{3,\mathrm{IM3L}}\ A^3 e^{j\phi_{a_3}} \\
+ \frac{|\,a_5\,|}{2^{5-1}} c_{5,\mathrm{IM3L}}\ A^5 e^{j\phi_{a_5}} \\
+ \ldots \Big] \;. \qquad (12)
\end{aligned}
$$

where $c_{n,k} = \sum_{l=0}^{\frac{n-3}{2}} \binom{n}{\frac{n-3}{2}-l,\, l+1,\, l,\, \frac{n+1}{2}-l}$ which represents the summation of all of the terms for each order that contribute to the frequency term k, here $\omega_k = 2\omega_1 - \omega_2$ [10]. From (12) it is clear that the effect of the phase shift in the linear sources is to introduce an additive phase term to that generated within the DUT. Therefore, the total relative phase at the DUT output at a particular power and frequency is

$$
\phi_y = \phi_s + \phi_{DUT} \;. \qquad (13)
$$

where $\phi_s = \Delta P_{dB,s} \cdot m_s(\omega_1)$. Similarly, the total phase at the output of the cancellation source is

$$
\phi_y = \phi_{meas} + \phi_c \;. \qquad (14)
$$

where ϕ_{meas} is the phase added by the cancellation source to achieve cancellation and $\phi_c = \Delta P_{dB,c} \cdot m_s(\omega_1)$ is the linear phase inserted by the cancellation source. Note the power terms in (13) and (14) are the power differences calculated from the small-signal reference power for both the stimulus sources and the cancellation source. Since the cancellation tone must be 180° out of phase with the desired frequency component at the DUT output, and the 0° reference cancellation phase was set at the small-signal power level, the actual phase of the frequency component under investigation is

$$
\phi_{DUT} = \phi_c + \phi_{meas} - \phi_s \;. \qquad (15)
$$

During the development of this measurement system, the same network arranged in Fig. 1 was also used to measure the traditional single-tone AM-AM, AM-PM data. It can be shown that the effect of the signal generator phase shift on the output phase of the DUT is identical to that of the two-tone case and thus the correction for the PM component can be extracted from the measured data as in (15).

IV. Measurement Results

The measurement system in Fig. 1 was used to measure the phase of the lower IM3 product during two-tone stimulus for a 5 W, connectorized, Mini–Circuits amplifier (ZHL-5W-1). The measurements were taken for a single tone separation of 10 kHz with $f_1 = 450$ MHz, suitable for the CDMA450 standard. The input tones' power was swept from -20 to 0 dBm, well past the input 1 dB compression point of -5 dBm. In addition to the IM3L phase measurement, an AM-AM, AM-PM measurement was made using the same instrumentation configuration with only one of the input sources supplying power. Accuracy of the phase measurement in this system is derived from the well-known equation for rejection such that in this case the worst case cancellation achieved for any data point was 28 dB, which corresponds to a maximum phase error (with assumed identical tone powers), of $\sim 2°$. The average cancellation achieved was 42 dB that results in a worst case phase error of $\sim 0.5°$.

The raw phase measurement of the lower IM3 product along with the corrected values using the calibration data is shown in Fig. 5. The phase of the IM3 products shows a different characteristic than typically assumed in that the phase is not constant at lower power input levels. This suggests that the nonlinear processes at work in the amplifier have memory components that shift the phase of the IM products, increasing this phase shift with increasing power. The result of the AM-AM, AM-PM single tone measurement is as expected, which aids in verifying the performance of the measurement setup, (Fig. 6).

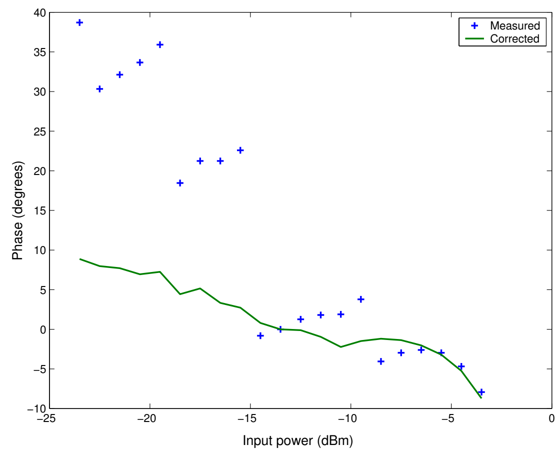

Fig. 5. Measured and corrected relative phase for IM3L component at DUT output.

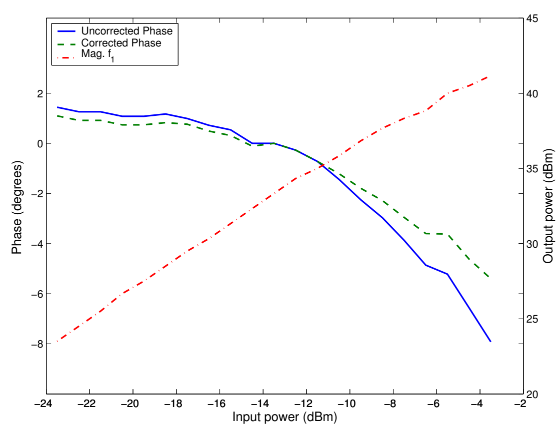

Fig. 6. AM-AM, AM-PM data with corrected phase measurement.

V. Conclusion

Here we have presented the development of a measurement system for determining the phase of the third-order intermodulation product of a device or system operating in its nonlinear region. The measurement system is based on the concept of phase measurement through a cancellation bridge used in existing techniques, although in our setup, no reference nonlinearity is required. Our technique uses only a small number of common microwave instruments and components, thereby simplifying the measurement and calibration procedure. Due to the general nature of the components used, this system can be used over a wide bandwidth of stimulus frequencies with only minor changes to the setup. Future work will leverage the measurement capabilities of the system for providing the necessary data for more accurate behavioral models of microwave devices and systems.

VI. Acknowledgments

This material is based upon work supported by the U.S. Army Research Office under grant number DAAD19-02-1-0153.

References

[1] T. Van den Broeck and Jan Verspecht, "Calibrated Vectorial Nonlinear-Network Analyzers," IEEE MTT-S Digest, vol. 2, pp. 1069-1072, May 1994.

[2] K. Remley, D. Williams, D. Schreurs, and J. Wood, "Simplifying and Interpreting Two-Tone Measurements," *IEEE Trans. on Microwave Theory and Techn.*, vol. 52, pp. 2576-2584, Nov. 2004.

[3] U. Lott, "A Method for Measuring Magnitude and Phase of Harmonics Generated in Nonlinear Microwave Two-Ports," IEEE MTT-S Digest, vol. 1, pp. 225-228, May 1998.

[4] P. Heymann, R. Doerner, and M. Rudolph, "Multiharmonic Generators for Relative Phase Calibration of Nonlinear Network Analyzers," *IEEE Trans. on Instrumentation and Measurement*, vol. 50, pp. 129-134, Feb. 2001.

[5] N. Suematsu, T. Shigematsu, Y. Iyama, and O. Ishida, "Transfer Characteristic of IM3 Relative Phase for a GaAs FET Amplifier," IEEE MTT-S Digest, vol. 2, pp. 901-904, June 1997.

[6] Y. Yang, J. Yi, J. Nam, B. Kim, and M. Park, "Measurement of Two-Tone Transer Characteristics of High-Power Amplifiers," *IEEE Trans. on Microwave Theory and Techn.*, vol. 49, pp. 568-571, Mar. 2001.

[7] J. H. K. Vuolevi, T. Rahkonen, J. P. A. Manninen, "Measurement Technique for Characterizing Memory Effects in RF Power Amplifiers," *IEEE Trans. on Microwave Theory and Techn.*, vol. 49, pp. 1383-1389, Aug. 2001.

[8] D. J. Williams, J. Leckey, and P. J. Tasker, "A Study of the Effect of Envelope Impedance on Intermodulation Asymmetry Using a Two-Tone Time Domain Measurement System," IEEE MTT-S Digest, vol. 3, pp. 1841-1844, June 2002.

[9] J. Dunsmore and D. Goldberg, "Novel Two-Tone Intermodulation Phase Measurement for Evaluating Amplifier Memory Effects," 33rd European Microwave Conference, vol. 1, pp. 235-238, Oct. 2003.

[10] K. Gharaibeh, and M. Steer, "Modeling Distortion in Multi-channel Communication Systems," *IEEE Trans. on Microwave Theory and Techn.*, May 2005.