A Distributed Coordination Mechanism for Shipment Consolidation

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September 22, 2010

Introduction

The consolidation of multiple shipments into a single truckload provides an alternative to transporting each shipment independently as either a single-shipment, point-to-point truckload (P2P TL) or via a less-than-truckload (LTL) carrier. The benefit of consolidation is the potential savings that may accrue from economies of scale in transportation when several smaller shipments are combined into a single load. In this research, a mechanism is developed for constructing consolidated shipment loads that uses only publically available information concerning each shipment along with simple standardized procedures to allocate the cost of transporting the load to the shipments in each load. (An early version of this research appeared in [1].) The advantage of using such a distributed consolidation mechanism is that each shipment can quickly and independently determine what its allocated portion of the transport charge will be, thus eliminating the need for a trusted center to determine the allocation and making it possible for each shipment to pre-determine its allocated charge during the planning stage when it is considering the possibility of joining various alternate loads that are available for consolidation. Although a central site is used to store information concerning available loads and to allocate, collect, and distribute the transport charge of each consolidated load, each shipment can independently verify all of the processing that occurs at the site because all of the information and procedures used in the processing are publically available.

The coordination mechanism is designed so that it can serve as almost cost-free means to enable all of the benefits of consolidation to be divided between the shippers, as opposed to other means of consolidation that share a significant portion of benefits with a third party as the cost of facilitating the consolidation. The benefit, or “savings,” associated with consolidation is the difference between sum of what the charges would be to transport each shipment independently and the charge to transport the shipments as a consolidated load. How the savings are allocated between the shippers, the carrier, and any other third party that helps facilitate the consolidation is based on both knowledge of the number of shipments available for consolidation and on the variability with respect to location and frequency of the shipments. A firm that generates a large number of shipments can perform its own consolidation and thereby capture all of the savings. A
firm that ships infrequently but on a regular basis can use a transportation exchange to capture most of the savings because it is able to commit to regular shipments across a lane and it is able to spread the one-time cost of using the exchange across many shipments, where the cost is primarily the time and effort associated with participating in the various auction mechanisms that occur on the exchange. A firm that infrequently ships between a variety of different origins and destinations typically uses a third party to facilitate consolidation, where such a consolidator keeps a portion of the savings as the cost of providing the service. In the case of small shippers with many one-time shipments to different destinations, the cost of using a consolidator can be significant because of the lack of information each shipper has concerning what other shipments that are available for consolidation. This increases the value of the service provided by the consolidator, and can result in the consolidator being able to keep most of the savings. In this research, each one-time shipment has access to all of the information concerning what other shipments are available for consolidation at different times in the future. It can use this information along with standardized procedures that are specified as part of the coordination mechanism to quickly determine what its expected charge would be if it were to join any of the available loads, and, once it joins a load, all of the consolidation savings are split between the shipments in each load. The only cost of facilitating this type of consolidation is the cost of running the servers used to implement the coordination mechanism.

**Coordination Mechanism**

In addition to just facilitating consolidation, the coordination mechanism is designed to encourage a shipment to post its availability for consolidation as early as possible, thereby allowing other shipments more opportunity to join its load, and to encourage a carrier to agree to transport a load as early as possible so that the carrier has more time to plan how to best transport the load. The first shipment to post its availability for consolidation is given the opportunity to determine the time at which the load will be transported, the transport rate for the entire load, and the minimum truck capacity. Subsequent shipments would join the putative consolidated load if their inclusion in the load results in a savings, and the time at which they join the load is used to order their priority with respect to determining the load’s routing sequence. The route sequence is determined by a standard procedure executed by the coordination mechanism. When joining a putative load, a new shipment provides a requested size. This requested size, along with the origin, destination, and density of the shipment, is used to determine its allocated charge. Although the actual charge is only collected from the new shipment at the time of route execution, the other shipments in the load can use the size of the new shipment to possibly alter their requested size since their allocated charge will likely change as a result of savings associated with the addition of the new shipment into the load.

At any time prior to the requested transport time of a load, a truck with minimum required capacity can agree transport the load and would be paid the sum of the allocated charge of each shipment in the load. In order to keep the scope of this paper limited to the just the essential elements needed to implement a distributed coordination mechanism, all of the issues regarding how the mechanism interacts with trucks are ignored. Instead, only the elements of the
mechanism needed for load formation prior to a truck agreeing to transport a load are discussed. The load is termed “scheduled” when a truck agrees to transport it, and is termed “executed” when the actual transport operation occurs. As an example of the issues associated with scheduling, any shipment should be able to request to be removed from the load without penalty any time prior to scheduling; but, between scheduling and execution, its removal would result in a decrease in the truck’s expected revenue from agreeing to transport the load and should be expected to incur a penalty charge equal to the lost revenue. Although these truck-related issues are important, they are independent of the mechanism needed for load formation prior to its scheduling.

The following basic procedures are needed for the coordination mechanism.

1. **Create New Load:** A shipment creates a new (unscheduled) load by specifying its desired size, public characteristics such as its density and origin and destination, and the following load parameters: a transport rate, minimum truck capacities, and the desired time when the transport of the load should be executed. These parameters will apply to all shipments that subsequently join the load. The load is then posted on the server.

2. **Route Sequencing:** The primary requirement for route sequencing is that the procedure uses an online algorithm so that the route can be constructed in a serial fashion as each shipment is added to the load. In addition, the procedure should be efficient because it will be used by each shipment to determine which load, among potentially many loads, it should consider joining.

3. **Determine Standard Transport Charge:** The primary requirements for determining transport charges are that a single standard model is used for shipments to determine both what their charge would be if they were transported as an independent, single-shipment load and as part of a consolidated load, and that the model only uses the public characteristics of each shipment to determine the charge.

4. **Allocate Transport Charge:** How the total transport charge associated with each consolidated load is allocated to each shipment in the load is the most important factor in enabling efficient shipment consolidation. The allocation procedure should base each shipment’s charge on the relative benefit to other shipments that it provides as a result of its presence in the consolidated load in order to promote overall efficiency.

5. **Add Shipment to Load:** A shipment can request to be added to an existing load by specifying its desired size and its public characteristics. All of the shipments in the load are then given opportunities to change their desired sizes and, when there is no further desired change, each shipment is given the opportunity to confirm the addition of the new shipment to the load. This allows any of the current shipments in the load to veto the addition of the new shipment, and insures that all of the shipments will benefit from the new shipment being added to the load.

6. **Remove Shipment from Load:** A shipment can request to be removed from an existing (unscheduled) load. As with the addition of a shipment, all of the remaining shipments in
the load are then given opportunities to change their desired sizes. Since the load is unscheduled, the shipment incurs no penalty as a result of its removal.

The primary procedural requirement for the coordination mechanism is the ability to add a shipment to a load. The transport rate, minimum truck capacities, shipment information, etc., are all determined outside of the mechanism, and the process used to determine the allocation of the transport charge includes as input the results from the process used to determine the transport charges and the route sequencing. Although allocation procedure is only used by the mechanism after the load is scheduled, each shipment can itself implement the procedure and use it as part of how it determines its desired size both when first joining a load and when subsequently queried by the mechanism to update their desired size as a result of other shipments joining the load.

**Three-Shipment Example**

An example of three shipments is illustrated in Figure 1. As independent shipments, each has a different origin and destination. Shipment 1 travels 300 miles from its origin at location 1 to its destination at location 2, shipment 2 travels 250 miles from location 3 to 4, and shipment 3 travels 275 miles from location 5 to 6.

![Figure 1. Three-shipment example.](image)

**Figure 2. Load sequence.**
Create New Load

A shipment creates a new load by specifying its desired weight (size), $q$, and other public parameters such as its density, $s$, and its origin (beginning) and destination (ending) locations, all of which are assumed to be verifiable by any shipment that subsequently joins the load. The density is used to determine the shipment’s cube requirements, and is used in place of directly specifying the cube because the shipment’s desired size can change in response to the addition of other shipments to the load. Each location is identified by a unique index that can be used to reference the place where a truck used to transport the shipment can pick up (load) or deliver (unload) the shipment, and to determine the actual road distance, $d$, between both locations. It is possible that multiple shipments might share the same location, offering the possibility to allocate a single minimum charge between each of the shipments at that location.

In addition to specifying its size and public parameters, the first shipment specifies a set of fixed parameters that determine the overall characteristics of the resulting consolidated load. Subsequent shipments joining the load are only able to specify their size. As a result, the first
shipment has the opportunity set the parameters to its advantage; but this advantage is mitigated by the need for the first shipment to set values for the load that are attractive to other shipments so that will want to join the load and to the trucks so that they will be willing to consider transporting the load. The load parameters specified are the transport rate, \( r \), the minimum truck weight, \( K_{wt} \), the minimum truck cube, \( K_{cu} \), and the desired time, \( t \), when the transport of the load should be executed. The rate \( r \) represents the cost per mile that will be paid by the consolidated load to the truck for its transport, subject to the truck providing a minimum capacity \( K_{wt} \) and \( K_{cu} \) for the load. On average, \( r \) should reflect the truck’s revenue per loaded-mile; in 2004, the average \( r \) is the U.S. was $2 per mile [2]. If the actual charge that another shipment would incur was significantly greater than \( r \), then it can always initiate the formation of an alternate load using a value for \( r \) that better reflects its actual charge.

**Route Sequencing**

The standard procedure to determine the minimum distance route sequence takes the sequence \( L = (y_1, \ldots, y_n) \) of \( n \) shipments in a load as input and outputs a \( 2n \)-element route sequence \( R = (z_1, \ldots, z_{2n}) \), representing the sequence in which the loading and unloading of each shipment occurs, so as to minimize the total distance travel from the loading of the first shipment to the unloading of the last shipment in the route. The first occurrence of \( z_j = y_i \) in \( R \) indicates the loading of shipment \( y_i \), and the second occurrence of \( z_k = y_i \) indicates its unloading, where \( j < k \). Each element \( z_i \) of route sequence \( R \) has a corresponding location \( x_i, X = (x_1, \ldots, x_{2n}) \), representing either the origin or destination location of the shipment. An \( 2n \)-element route \( R \) has \( 2n - 1 \) segments, where each segment consists of a consecutive pair of elements in \( R, (z_i, z_{i+1}) \).

The total distance of \( R \) is the sum of the distance of each of its segments. Assuming that the distances \( d_{ij} \) between all pairs of locations \( i \) and \( j \) are publicly known,

\[
d(R) = \sum_{i=1}^{2n-1} d_{x_i, x_{i+1}}
\]

represents the distance of route \( R \). For example, referring to Figure 1, the route sequence \((3,1,2,2,1,3)\) would correspond to travel from location 5 after loading shipment 3 to location 1 to load shipment 1, to location 3 to load 2, to 4 to unload 2, to 2 to unload 1, and to 6 to unload 3. The total distance of the route is 430, where \( d_{5,1} = 60 \), etc.

The route sequencing problem is equivalent to the traveling salesman problem and simple heuristics can be used to provide reasonably good solutions. In order to make the route sequencing procedure especially simple and fast, an online procedure is used that takes as input only the current route sequence and the shipment to be added to the sequence. This is opposed to a more time-consuming offline procedure that would use all of the shipments to construct the route sequence. While an offline procedure can sometimes produce better results and would make the resulting route independent of the order in which shipments are added to the route, the extra time required is not deemed worth the effort since the goal of the coordination mechanism is to allow many different potential route sequences to be considered with a reasonable amount of effort. Also, shipments are added to a route so as to minimize its total distance. This criterion is used, as opposed to minimizing the total transport charge of the route, because the transport
charge can be a function of the shipment size and could require that the route be re-determined each time there was a change in size.

**procedure sequenceRoute**(\(y_i \in L\))
\[
R = (y_1, y_1)
\]
for \(i = 2, \ldots, |L|\)
\[
R = \text{minDistanceInsert}(y_i, R)
\]
\[
R = \text{twoOptImprove}(R)
\]
endfor
return \(R\)

**subprocedure minDistanceInsert**(\(y, z_i \in R\))
\[
d_R = d(R)
\]
for \(i = 1, \ldots, |R| + 1\), for \(j = 1, \ldots, |R| + 1\)
\[
R' = (z_1, \ldots, z_i-1, y, z_i, \ldots, z_{j-1}, y, z_{j}, \ldots, z_{|R|})
\]
if \(d(R') < d_R\), \(d_R = d(R')\), \(R = R'\), endif
endfor, endfor
return \(R\)

**subprocedure twoOptImprove**(\(z_i \in R\))
\[
d_R = d(R)
\]
repeat
\[
done = \text{true}, \ i = 1, \ j = 2
\]
while \(done\) and \(i < |R|\)
\[
\text{while} \ done \text{ and } j < |R| + 1
\]
\[
R' = (z_1, \ldots, z_{i-1}, \text{reverseSequence}(z_i, \ldots, z_j), z_{j+1}, \ldots, z_{|R|})
\]
if \(d(R') < d_R\)
\[
d_R = d(R')\), \(R = R'\), \(done = \text{false}
\]
endif
\[
j = j + 1
\]
endwhile
\[
i = i + 1, \ j = i + 1
\]
endwhile
until \(done = \text{true}\)
return \(R\)

**Figure 4. Route sequencing procedure.**

Given a load sequence \(L\), the procedure **sequenceRoute** shown in Figure 4 is used to construct a route \(R\). Each shipment \(y\) in \(L\) is added to \(R\) using the subprocedure **minDistanceInsert** that determines where the loading and unloading of the shipment can be added with the minimum increase in distance. In **minDistanceInsert**, \(y\) is inserted between subsequences of \(R\), where any subsequence \((z_i, \ldots, z_j)\) where \(i > j\) corresponds to the empty sequence \((\ )\); thus, for example, when \(ij = 1\), \((z_1, z_0, y, z_1, z_0, y, z_1, \ldots, z_{|R|})\) reduces to \((y, y, z_1, \ldots, z_{|R|})\), a route where the loading and unloading of the new shipment occurs before any of the other shipments in the sequence. After each shipment has been inserted into \(R\), the pairwise exchange subprocedure
twoOptImprove is used to possibly improve the route by replacing nonconsecutive pairs of segments \((z_{i-1}, z_i)\) and \((z_j, z_{j+1})\) in \(R\) with segments \((z_{i-1}, z_j)\) and \((z_i, z_{j+1})\), and reversing the subsequence \((z_i, \ldots, z_j)\) until a reduction in distance is found; if so, the \(R\) is replaced with the improved route and the process restarts, continuing until no improvement is found for all pairs of segments. The total distance of each route created by the procedure is dependent on the order of the shipment sequence \(L\). For example, referring to the three shipments shown in Figure 1, there are six different possible sequences \(L\).

**Standard Transport Charge**

A single standard model is used to determine the transport charge for both independent, single-shipment loads and consolidated loads, and that the model only uses the public characteristics of each shipment and load to determine the charge. The specific details regarding how costs are represented in the model can differ depending on the type of transport operations being considered, with different models producing different results with respect to the performance of the consolidation mechanism. In particular, the model should define a transport charge \(c^0\) that can be used to determine what the nominal charge would be for each shipment if it was transported as an independent shipment of size \(q\) and a transport charge \(c\) that can be used to determine what the actual total charge would be to transport the entire load \(L\) of size \(Q = (q_1, \ldots, q_{|L|})\).

The focus of this paper is on just the shipper side of a single transport leg, where each leg of a shipment is defined as its transport on a single vehicle, and, in particular, on transport via the most common tractor and 48–53 ft dry van trailer used in the U.S., with a maximum weight capacity of 25 tons and an effective cube capacity of 2,750 ft\(^3\), which is about 80% of a trailer’s physical cube capacity. In the transport charge model, both the consolidated load and the independent P2P TL shipments are assumed to be transported using this type of vehicle, and the charge for using LTL to transport a shipment is made commensurate with the P2P charge so that a full range of shipment sizes can be considered using the mechanism. A different model would be needed to represent the charge associated with, for example, pickup and delivery operations via the Class 4 vans used by parcel delivery companies; in particular, the minimum transport charge and maximum shipment size would be much smaller compared to those of a long-haul tractor-trailer.

Although, in practice, every shipper can negotiate with a P2P TL or LTL carrier to determine a unique charge for each independent shipment, this would not be suitable for use in a direct coordination mechanism for one-time shipments because the result of the negotiation can depend on a number of factors that involve private information that is not verifiable (e.g., the bargaining power of the shipper versus the carrier). As a result, some type of time-consuming auction procedure would be needed to discover this private information so that it could not be used to manipulate the allocation procedure; but this would make it difficult to use the coordination mechanism to determine the charge for consolidated loads that will occur in the future. Instead, a simple standardized model is used to determine the transport charge that requires only public load and shipment information.
Independent Charge

In order to be able to be considered together, the charges for P2P TL and LTL transport must be made commensurate with each other and with the charge for transporting the consolidated load. The ratio, $k$, of the load’s full-capacity-equivalent transport rate $r$ to the rate for a specific year ($2$ per mile, in 2004) is used to adjust the TL and LTL minimum charge values and the LTL Producer Price Index (PPI) value to reflect the change in these values from the year 2004, and serves to maintain the link between all three values. Since the 2004 rate of $2$ per mile applies to truck trailers with a maximum weight capacity of 25 tons and an effective cube capacity of 2,750 ft$^3$, while the load’s requested $K_{wt}$ and $K_{cu}$ might be less than 25 tons and 2,750 ft$^3$, respectively, the load’s implied requested utilization of

$$u = \min \left\{ 1, \max \left\{ \frac{K_{wt}}{25}, \frac{K_{cu}}{2750} \right\} \right\}$$

is used to possibly inflate $r$ to reflect the load’s full-capacity-equivalent transport rate so that it is commensurate with the $2$ per mile rate:

$$k = \frac{r}{u}.$$

The following equation [2] can be used to estimate the average 2004 LTL rate (in $\$\text{ per ton-mile}$) given the size $q$ (in tons), density $s$ (in lb/ft$^3$), and actual road distance $d$ (in miles) of the shipment:

$$r_{LTL}(q,s,d) = 104.2 \left[ \frac{\frac{s^2}{8} + 14}{\left( \frac{1}{q^7} \frac{15}{d^{29}} - \frac{7}{2} \right) \left( s^2 + 2s + 14 \right)} \right], \quad (1)$$

where (1) is not valid for $q < 150$ lb or $d < 37$, and 104.2 is the LTL PPI value for 2004. Using the same dataset used in [2] along with $k$, the average minimum charge in for an LTL shipment can be estimated as

$$c_{LTL}^{\min} = k \left( 45 + \frac{28}{d^{19}} \right). \quad (2)$$

The minimum charge in (2) is assumed to be associated primarily with the fixed costs of loading or unloading a shipment, and the largest portion of these fixed costs is assumed to be the associated with the time required by a truck to position itself at a location prior to the actual loading/unloading operation. The minimum charge is a function of the distance of the shipment because the number of loading/unloading operations increases as the number of line-haul legs increase as a shipment moves through an LTL carrier’s terminal network. Since a P2P TL shipment requires only a single loading and unloading, the distance-related term in (2) can be dropped to get the minimum charge for TL:

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Using $k$ to adjust (1) to reflect the full-capacity-equivalent PPI for LTL, the combined P2P TL and LTL independent transport charge (in \$) is

$$c^0 = \min \left[ \max \left\{ r d, c^\text{min}_{TL} \right\}, \max \left\{ k r_{TL} (q, s, d) q d, c^\text{min}_{LTL} \right\} \right],$$

(4)

where it is assumed, for P2P TL, that any portion of the load exceeding the maximum requested payload,

$$q^\text{max} = \min \left\{ K_w r, \frac{s K_c r}{2000} \right\},$$

would still be transported at the rate $r$ (in practice, any shipment requesting a size even close to the maximum payload would not be a candidate for consolidation, and no truck would likely accept a shipment if it’s size exceeded the maximum payload). For a given $r$, (4) provides a simple means to determine a nominal independent shipment charge.

Figure 5. Transport charges for shipment 1 of Figure 1.

The top solid curve in Figure 5 shows an example of what the independent charges would be for a range of sizes for the transport of shipment 1 of Figure 1, given $s = 10$ lb/ft$^3$, $d = 300$ miles, $r = $2 per mile, $K_w = 25$ tons, and $K_{cu} = 2750$ ft$^3$. (For illustration purposes, the range of shipment sizes along the bottom axis in Figure 5 is not drawn at a constant scale.) At less than 150 lb (or 150/2000 tons), the charge equals a minimum charge of $c^\text{min}_{LTL} = $48 (if $d < 22.5$, then the charge would be $c^\text{min}_{TL} = $45, due to (1) not being defined for $d < 37$). From 150 lb to 1.65 tons, transporting the shipment LTL is less costly than P2P TL; beyond this, P2P TL is less costly until $q^\text{max} = 13.37$ tons is reached, which is the maximum payload for the shipment. The region of below the independent charge curve corresponds to what the shipment’s allocated charge could be as a result of the shipment being part of a consolidated load. The bottom dashed curve corresponds to the likely lower bound on the allocated charge that could occur if the consolidated
load used all of the truck trailer’s available capacity so that each shipment is only allocated a charge equal to its portion of capacity.

Figure 6 illustrates the effect of different combinations of load parameters $r$, $K_{wl}$ and $K_{cw}$ on the resulting shape of the transport charge curve. In Figure 6(a), which is same as Figure 5, $2$ per mile is offered for a truck’s full capacity ($u = 1$, $k = 1$), while in (b), $1$ per mile is offered for half of the truck’s capacity ($u = 0.5$, $k = 1$). In Figure 6(c), only $1$ per mile is offered for a truck’s full capacity ($u = 1$, $k = 0.5$), while in (b), $2$ per mile is offered for only half of the truck’s capacity ($u = 0.5$, $k = 2$).

Consolidated Charge

Since each consolidated load is transported as a truckload, its charge can be made commensurate with the independent charges of each of the shipments comprising the consolidated load by using the same load parameters and viewing each different mix of shipments in the consolidated load as a single “aggregate shipment” for which a transport charge can be determined. Given a consolidated load $L$ and corresponding route and location sequences $R$ and $X$, respectively, its minimum charge is determined by taking the sum of half of the P2P TL minimum charge as determined in (3) for each distinct sequence of locations in $X$. As assumed for independent shipments, the minimum charge for a consolidated load represents a fixed cost for the truck to position itself at each location and does not depend on the number or size of the shipments being loaded and/or unloaded at each location. The minimum charge at each location is then allocated to each of the shipment at the located. As a result, for a small shipment, there is a benefit for using common locations for each leg of its transport.
For each shipment $y_i$ in $L$, its allocated minimum charge is

$$c_{i}^{\text{min}} = \left( \frac{1}{m_i^b} + \frac{1}{m_i^e} \right) c_{TL}^{\text{min}},$$

(5)

where $m_i^b$ and $m_i^e$ are the number of common consecutive elements in $X$ containing the beginning and ending locations, respectively, of $y_i$. The minimum charge for the entire load is

$$c^{\text{min}} = \sum_{i=1}^{L} c_{i}^{\text{min}}.$$

(6)

For each of the $|R| - 1$ segments in $R$, let $L_i$ represent the set of shipments onboard the truck for segment $i$ of the route. The transport charge for consolidated load $L$ is then

$$c_L = \max_{i=1, \ldots, |R|-1} \left\{ \max \left[ c_{i}^{\text{min}}, \min \{ r d_R, k r_{LTL}(q_L, s_L, d_R) q_L d_R \} \right] \right\},$$

(7)

where $d_R$ is the distance of the entire route $R$, $q_L$ is the sum of the size $q_{y_j}$ of each shipment $y_j$ in $L_i$, and, given the density $s_{y_j}$ of each shipment,

$$s_{L_i} = \frac{q_L}{\sum_{j=1}^{L} q_{y_j}}$$

is the aggregate density of all of the shipments in the segment.

Each route has one or more different mixes of shipments, where each distinct mix corresponds to a unique set of shipments in a segment that is not contained in the set of shipments in any other segment of the route. As an alternative to determining the charge for each segment in the route, only the charge of each mix need be determined. The resulting savings in computation is somewhat offset be the cost for determining the mixes in a route.

Figure 7. Five-shipment example.
In Figure 7, five shipments are consolidated given the load sequence \((1, \ldots, 5)\). The resulting route and location sequences are \((3, 2, 5, 5, 1, 4, 3, 1, 2, 4)\) and \((4, 3, 7, 1, 1, 6, 5, 2, 2, 2)\). Given \(k = 1\), the allocated minimum charges for shipments 1 through 5 are 18.75, 30, 45, 30, and 33.75, respectively.

**Allocate Transport Charge**

The allocation of the total transport charge associated with each consolidated load to each shipment in the load should be determined by using an approximation of the Shapley value of the savings in cost each shipment provides the load as compared to its independent transport. An approximation is used because determining the complete Shapley value would involve determining the expected savings associated with all of the possible subsets of shipments, which requires exponential complexity, and, using the route sequencing procedure as specified in Figure 4, the value of the savings of each subset of shipments is sequence dependent. Instead, only the sequence-dependent savings associated with all of the shipments and each pair of shipments is used as an approximation to the complete Shapley value.

The savings are determined by the difference between the sum of the nominal independent shipment charges associated with each of the \(n\) shipments in the load \(L\) and the charge for transporting the consolidated load:

\[
c_L^{\text{sav}} = \sum_{i=1}^{n} c_i^0 - c_L.
\]

Each shipment \(y_i\) in \(L\) is then allocated a portion \(c_i^{\text{sav}}\) (defined in (15), below) of the savings, such that \(\sum_{i=1}^{n} c_i^{\text{sav}} = c_L^{\text{sav}}\). Each shipment’s allocated savings is then subtracted from its independent charge to determine its allocated charge. In a similar manner, the pairwise savings for shipments \(i\) and \(j\) is

\[
c_{ij}^{\text{sav}} = c_i^0 + c_j^0 - c_{(i,j)}
\]

where \(c_{(i,j)}\) is the transport charge for the consolidated load \((i, j)\).

The complete Shapley value for shipment \(i\) can be defined as follows \([3]\):

\[
\alpha_i = \sum_{0 \leq m \leq n-1} \frac{m!(n-m-1)!}{n!} \sum_{\substack{M \subset N \setminus \{i\} \mid |M| = m}} (\sigma_{M \cup \{i\}} - \sigma_M),
\]

where \(N\) is the (unordered) set of \(n\) shipments in the load (as compared to the ordered sequence \(L\)), \(M\) the subsets of shipments, and \(\sigma_M\) is the non-sequence-dependent savings associated with \(M\), where \(\sigma_{\emptyset} = 0\). For \(n = 3\), (10) for shipment 1 can be written as a combination of its three-way \((\sigma_M)\), pairwise \((\sigma_{ij})\), and single-shipment \((\sigma_i)\) savings ([3], p. 111)
\[
\alpha_1 = \frac{\sigma_1}{3} + \frac{1}{6}\left[ (\sigma_{1,2} - \sigma_2) + (\sigma_{1,3} - \sigma_3) \right] + \frac{1}{3}(\sigma_N - \sigma_{2,3}) \\
= \frac{\sigma_N}{3} + \frac{1}{6}(\sigma_{1,2} + \sigma_{1,3} - 2\sigma_{2,3}) + \frac{1}{6}(2\sigma_1 - \sigma_2 - \sigma_3) \\
= \frac{\sigma_N}{3} + \frac{1}{6}(\sigma_{1,2} + \sigma_{1,3} - 2\sigma_{2,3}),
\]

(11)

where all of the single-shipment savings are zero. This last form suggests that, while exact for \( n \leq 3 \), an approximate Shapley value for \( n > 3 \) can be determined by viewing the net pairwise savings of a shipment \( i \), \( \Delta\sigma_i \), as an adjustment to an equal allocation of the total savings. The ratio of total savings to pairwise savings (1/3 to 1/6 in (11)) can be determined for \( n > 3 \) by replacing \( m \) in (10) first with \( n - 1 \) (corresponds to the savings associated with all shipments), so that \( m!(n-m-1)!/n! \) reduces to 1/n, and then with 1 (corresponding to all pairwise savings), so that \( m!(n-m-1)!/n! \) reduces to 1/(n(n-1)), resulting in an allocated savings for shipment \( i \) of

\[
c_i^{\text{sav}} = \frac{\sigma_N}{n} + \frac{1}{n(n-1)}(\Delta\sigma_i).
\]

(12)

In order for the approximation to be a valid allocation, the \( c_i^{\text{sav}} \) for all \( n \) shipments should sum to \( \sigma_N \), which will be used to represent \( c_L^{\text{sav}} \). The first term in (12) sums to \( \sigma_N \) as long as the net pairwise savings for all shipments (the second term in \( \sigma_N \)) sums to zero:

\[
\sum_{i=1}^{n} (\Delta\sigma_i) = \sum_{i=1}^{n} \left( \kappa \sum_{j=1}^{n} \sigma_{ij} - 2 \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \sigma_{jk} \right)
\]

\[
= \sum_{i=1}^{n} \left[ \kappa \sum_{j=1}^{n} \sigma_{ij} - \left( \sum_{j=1}^{n} \sum_{k=1}^{n} \sigma_{jk} - 2 \sum_{j=1}^{n} \sigma_{ij} \right) \right]
\]

\[
= \kappa \sum_{j=1}^{n} \sum_{j=1}^{n} \sigma_{ij} - n \sum_{j=1}^{n} \sum_{k=1}^{n} \sigma_{jk} + 2 \sum_{j=1}^{n} \sum_{k=1}^{n} \sigma_{ij} = 0,
\]

(13)

(14)

where use is made of the fact that all \( \sigma_{ij} = \sigma_{ji} \) and \( \sigma_{ii} = 0 \). Solving (14) yields \( \kappa = n - 2 \). Using \( \kappa \) and the term inside the summation in (13) and letting the total savings \( \sigma_N \equiv c_L^{\text{sav}} \) and all pairwise savings \( \sigma_{ij} \equiv \left( c_{ij}^{\text{sav}} + c_{ji}^{\text{sav}} \right)/2 \), (12) reduces to

\[
c_i^{\text{sav}} = \frac{c_L^{\text{sav}}}{n} + \frac{1}{n(n-1)} \left[ (n-2) \sum_{j=1}^{n} \frac{c_{ij}^{\text{sav}} + c_{ji}^{\text{sav}}}{2} - \left( \sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk}^{\text{sav}} - \sum_{j=1}^{n} \left( c_{ij}^{\text{sav}} + c_{ji}^{\text{sav}} \right) \right) \right]
\]

\[
= \frac{c_L^{\text{sav}}}{n} + \frac{1}{n(n-1)} \left( n \sum_{j=1}^{n} \frac{c_{ij}^{\text{sav}} + c_{ji}^{\text{sav}}}{2} - \sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk}^{\text{sav}} \right)
\]

\[
= \frac{c_L^{\text{sav}}}{n} + \frac{1}{n-1} \sum_{j=1}^{n} \frac{c_{ij}^{\text{sav}} + c_{ji}^{\text{sav}}}{2} - \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk}^{\text{sav}}
\]

(15)
where \( \sigma_{ij} \equiv \left( \frac{c_{ij}^{\text{sav}} + c_{ji}^{\text{sav}}}{2} \right) \), instead of just \( c_{ij}^{\text{sav}} \), in case not all \( c_{ij}^{\text{sav}} = c_{ji}^{\text{sav}} \).

\[
\text{ procedure } \text{allocTransCharge}\left(\left[ c_i^0 \right]_{i \in L}, \left[ c_i^{\text{sav}} \right]_{i \in L}, \left[ c_i^{\text{min}} \right]_{i \in L}\right)
\]

\[
\text{ for } i = 1, \ldots, n, \ c_i = c_i^0 - c_i^{\text{sav}}, \ \text{ endfor}
\]

\[
\text{ repeat}
\]

\[
c^- = \sum_{i=1}^{n} \max \left\{ 0, c_i^{\text{min}} - c_i \right\}
\]

\[
\text{ if } c^- > 0
\]

\[
c^+ = \sum_{i=1}^{n} c_i - c^-
\]

\[
\text{ for } i = 1, \ldots, n
\]

\[
\text{ if } c_i > c_i^{\text{min}}, \ c_i = c_i - c^- \frac{c_i^0}{c^-}, \ \text{ else, } \ c_i = c_i^{\text{min}}, \ \text{ endif}
\]

\[
\text{ endfor}
\]

\[
\text{ endif}
\]

\[
\text{ until } c^- = 0
\]

\[
\text{ return } \left[ c_i \right]_{i \in L}
\]

Figure 8. Allocate transport charge procedure.

Given the independent charge \( c_i^0 \) (as defined in (4)), the allocated savings \( c_i^{\text{sav}} \) (as defined in (15)), and the allocated minimum charge \( c_i^{\text{min}} \) (as defined in (5)) for each of the \( n \) shipments in the load \( L \), allocTransCharge returns the allocated transport charge \( c_i \) of each shipment, such that all \( c_i^{\text{min}} \leq c_i \) and \( \sum_{i=1}^{n} c_i = c_L \). Since it is possible that, when a shipment’s allocated savings is subtracted from its independent charge, its allocated charge might be less its allocated minimum charge \( c_i^{\text{min}} \), the procedure allocTransCharge listed in Figure 8 is used to ensure that each shipment’s allocated charge at least equal its minimum charge. The reason that each allocated charge should at least equal its minimum charge is that the minimum charge represents the portion of the total charge for the load that is directly attractable to each shipment, and that any charge less that this would represent a subsidy from the other shipments. In the procedure, the amount that a shipment’s initial allocated charge falls below its minimum charge is subtracted from the other shipment charges for those shipments that exceed their minimum charges. The amount subtracted is based each shipment’s charge as a fraction of the total charge.
procedure addShmtToLoad \( y'; y_i \in L \)
\[
L' = (y_1, \ldots, y_{|L|}, y')
\]
for \( i = 1, \ldots, |L| \), \( q_i = get_q(i; L) \), endfor, \( q_{|L|} = 0 \)
done = false, isAdded = false, \( i = |L'| \)
repeat
\[
q'_i = \text{detSize}(i; L'; q'_1, \ldots, q'_{|L'|})
\]
if \( q'_i = 0 \)
    return isAdded
elseif all \( (|q'_i - q_i| < tol) \) = true
    for \( j = |L'|, \ldots, 1 \)
        if \( \text{confirmSize}(j; L'; q'_1, \ldots, q'_{|L'|}) = \) false, return isAdded, endif
    endfor
    done = true
else
    \( q_i = q'_i \)
    if \( i > 1 \), \( i = i - 1 \), else \( i = |L'| \), endif
endif
until done = true
post( \( L'; q'_1, \ldots, q'_{|L'|} \) )
return isAdded

Figure 9. Add shipment to load procedure.

Add Shipment to Load

The procedure \( \text{addShmtToLoad} \) listed in Figure 9 is used to add a shipment \( y' \) to a load \( L \). Starting with \( y' \), each shipment determines its desired size in the load through a call by \( \text{addShmtToLoad} \) to that shipment’s \( \text{detSize} \) procedure, where a returned size of zero is used to indicate that the addition of \( y' \) to \( L \) should not occur (a veto). All of the shipments continue to be polled for their desired size in the load until there has been no change in size between subsequent pollings for all shipments. No change is indicated by the difference in sizes falling below a tolerance \( tol \) (e.g., \( tol = 1/2000 \) would correspond to one pound, for sizes in tons). After polling, each shipment is given the opportunity to confirm the addition of the new shipment through a call by \( \text{addShmtToLoad} \) to that shipment’s \( \text{confirmSize} \) procedure, where a returned value of false indicates a veto. The ability to veto the addition of any new shipment to a load insures that each shipment will benefit. The initial size of each shipment is retrieved from the posted load information using a \( \text{get} \) procedure, and the final load sequence and shipment sizes is made available as public data using a \( \text{post} \) procedure.
The computational effort required to operate the coordination mechanism can be kept to a minimum since most of the effort can be distributed to each shipment’s $detSize$ and $confirmSize$ procedures. The $addShmtToLoad$ procedure can be implemented very efficiently because it does not require sequencing of each new shipment into a route; instead, the sequencing can be performed within each shipment’s $detSize$ and $confirmSize$ procedures.

**Remove Shipment from Load**

The procedure $removeShmtFromLoad$ listed in Figure 10 is used to remove a shipment $y^0$ from a load $L$. After $y^0$ has been removed, the $addShmtToLoad$ procedure is called to add each of the remaining shipments into one or more loads $L^1,...,L^i$ that replace $L$, each with the same fixed load parameters as $L$. Multiple loads can occur whenever the removal of $y^0$ results in the benefit associated with remaining as a single load falling below the benefit associated with the next best alternative for at least one of the remaining shipments.

```
procedure removeShmtFromLoad\( y^0; y_i \in L \) 
L = L \setminus y^0, \ i = 0 
while |L| > 0 
    i = i + 1, \ L^i = y_i, \ L' = L 
    while |L'| > 0 
        if $addShmtToLoad\( y^i_1; L^i \) = true$, \ L = L \setminus y^i_1, \ endif 
        L' = L' \setminus y^i_1, 
    endwhile 
endwhile
```

**Figure 10. Remove shipment from load procedure.**

**Shipment Procedures Required by Mechanism**

Each shipment using the mechanism is required to implement two procedures, $detSize$ and $confirmSize$, that are called by the $addShmtToLoad$ procedure (see Figure 9) of the distributed coordination mechanism. From the perspective of a shipper using the mechanism to determine whether it should join a consolidated load, a distinction can be made between the decision processes used for a one-time shipment and shipments that will occur repeatedly. It is straightforward to a shipment to use the mechanism for consolidation if it is a one-time occurrence and its size is fixed: every time its $detSize$ procedure is called by the mechanism, it can return either that fixed size or a value of zero if being in that load is not its lowest cost option. For periodic shipments, where there is flexibility with respect to when the shipments can occur because the product being transported can be held as inventory at the origin and/or the destination, the process that a shipment uses to return its size in its $detSize$ procedure depends on the relative cost of transport versus the cost of carrying inventory, as represented by its total
logistics cost (TLC). There are many different types of procedures that could be used to make these decisions, each of varying degrees of complexity and effectiveness. The procedures described in this section are an attempt to provide the simplest possible procedures that are still effective with respect to minimizing each shipment’s TLC over multiple shipments over an extended period of time.

Total Logistics Cost Criterion
To help formulate the decision problem for a periodic shipment, it will be assumed that all costs are represented by its total logistics cost (TLC) and that the demand rate \( f \) (in tons per year) estimated for the product being shipped will remain constant until at least the completion of the next shipment interval, at which time the size decision can be revised. The decision process used by the shipment is myopic in the sense that estimated future demand changes are not used. The TLC for a shipment is sum of the total transport and cycle inventory costs over some period of time, and can be defined as a function of the per-shipment transport charge \( c \) and the shipment size \( q \):

\[
TLC(c,q) = c \frac{f}{q} + q \nu,
\]

(16)

where \( \nu \) (in $/ton-yr) is the cost of holding one ton of inventory for one year shipment value. It is assumed that pipeline inventory costs incurred while the shipment is in transit can be ignored in the decision process because the one-day to one-week range of transit times associated with the different alternatives is not significant compared to the impact of one-day to multi-year range of shipment intervals on cycle inventory costs.

Size-Determination Procedure
As an independent shipment \( i \), the size that minimizes (16) can determined by making the transport charge in (4) a function of shipment size and then solving for

\[
q^0_i = \arg \min_q TLC(c^0_i(q), q).
\]

The TLC at \( q^0_i \), \( TLC^0_i = TLC(c^0_i(q^0_i), q^0_i) \), provides an upper bound on the TLC that shipment \( i \) would accept as part of a consolidated load.

In order for shipment \( i \) to be able to determine its TLC as part of a consolidated load \( L \), the shipment’s local implementation of the allocation procedure \( allocTransCharge \) (see Figure 8) is used to compute its expected transport charge \( c_i \). Given the sequence \( L \), all of the load and shipment parameters used in the allocation procedure are known to shipment \( i \) except what the final sizes will be for all of the shipments in the load. Defining \( \hat{q}^{j-i}_j(q_i) \) as the estimated size for each shipment \( j \) in \( L \setminus i \) as a function of \( q_i \), shipment \( i \)'s allocated portion of the transport charge for \( L \) can be represented as

\[
c_i\left(q_i, \left[\hat{q}^{j-i}_j(q_i)\right]_{j \in L \setminus i}\right),
\]

where \( q_i \) and each \( \hat{q}^{j-i}_j(q_i) \) are used to determine the inputs \( c^0 \) and \( c^{\text{sav}} \) to \( allocTransCharge \) (\( c^{\text{min}} \) is not a function of size), and shipment
i’s portion of the allocated charge, $c_i$, is returned. The $\text{detSize}$ procedure for shipment $i$ can then return

$$q_i^* = \arg \min_q TLC\left(c_i, \left[\hat{q}_j^i(q_j)\right]_{j \in L_i}, q\right)$$

(17)

if

$$TLC\left(c_i\left(q_i^*, \left[\hat{q}_j^i(q_j^*)\right]_{j \in L_i}\right), q_i^*\right) \leq TLC_i^0;$$

(18)

and $q_i^* = 0$, otherwise.

In (17), all that is needed to determine $q_i^*$, and thereby implement $\text{detSize}$, is to estimate each $\hat{q}_j^i(q_i)$. One simple estimate is just the current size, as posted, of each shipment in $L$: $\hat{q}_j^i(q_i) = q_j^i$. Unfortunately, this can result in poor performance, and is best illustrated by an example. Given shipment 1 of Figure 1 (with the same public parameters $s = 10$ lb/ft$^3$, $d = 300$ miles, $r =$ $2$ per mile, $K_{wt} = 25$ tons, and $K_{cu} = 2750$ ft$^3$ that were used in Figure 5 to plot its independent transport charges), assume that its private data is $f = 3$ tons per year and that $v = $ $450$ per ton-year, and that a second identical shipment, shipment 2, is available. As two independent LTL shipments, $q_1^0 = q_2^0 = 0.5282$ tons, $c_1^0 = c_2^0 = $ $234.16$, $TLC_1^0 = TLC_2^0 = $ $1,568$ per year, and, combined, $TLC_1^0 + TLC_2^0 = $ $3,135$ per year. Since the shipments are identical, the minimum TLC for the equivalent single aggregate shipment is $TLC_{1&2}^0 = $ $2,546$ per year, where all of the parameters are the same except $f_{1&2} = 6$ tons per year, and the aggregate shipment is P2P TL, with $q_{1&2}^0 = 2.8284$ tons and $c_{1&2}^0 = $ $600$ (see Figure 5).

<table>
<thead>
<tr>
<th>Calling Sequence</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returned $q_1$</td>
<td>0.5282</td>
<td>0.5282</td>
<td>0.3535</td>
<td>0.3535</td>
<td>0.3514</td>
<td>0.3514</td>
</tr>
<tr>
<td>Returned $q_2$</td>
<td>—</td>
<td>0.3354</td>
<td>0.3354</td>
<td>0.3511</td>
<td>0.3511</td>
<td>0.3514</td>
</tr>
<tr>
<td>$TLC_1 + TLC_2$</td>
<td>1,568</td>
<td>2,861</td>
<td>2,845</td>
<td>2,844</td>
<td>2,844</td>
<td>2,844</td>
</tr>
</tbody>
</table>

As can be seen in Table 1, the results of using the current posted size as the estimate for the other shipments in the load results in a total TLC of $2,845$, which exceeds the minimum possible TLC of $2,546$ when both are considered as a single aggregate shipment. The shipment sizes listed in the columns of the table indicate the sequence of values returned by the $\text{detSize}$ procedure of each shipment as it is called by $\text{addShmtToLoad}$, where the first column represents the use of $\text{addShmtToLoad}$ to create the load with shipment 1 as its first shipment. In addition, total transport charge for the consolidated load is $296.13$, which is well below the P2P TL charge and would likely require addition shipments to increase the charge to a level acceptable to a truck for transport.
A slightly more complicated means of estimating shipment size is to assume that each of the other shipments in $L$ will select a size so that, given your $q_i$, their per-mile independent transport charge equals your charge:

$$\hat{q}_j^{-i}(q_i) = \arg \min_q \left( \frac{c_j^0(q) - c_i^0(q_i)}{d_j - d_i} \right). \tag{19}$$

Table 2 shows the results of using (19) in (17), where the minimum possible TLC is quickly achieved. The total transport charge for the consolidated load is $600, which is equal to the P2P TL charge. The estimate in (19) only uses public data and procedures. If a shipment has other information regarding what the likely size would be for any of the other shipments in the load, then that could be incorporate into a proprietary implementation of its detSize procedure.

### Size-Confirmation Procedure

In addShmtToLoad, each shipment is given the opportunity to confirm the addition of the new shipment to the load through a call to that shipment’s confirmSize procedure, where a returned value of false indicates a veto. As with its detSize procedure, a shipment could possibly incorporate a variety of proprietary factors into its confirmation procedure. One simple implementation is to perform the following two checks:

First, if the shipment $i$ is the shipment being added to the load, then a check is made to see if its presence at a size of $q_i$ will cause the load’s requested weight or cube capacities, $K_{wt}$ and $K_{cu}$, to be exceeded, and, if so, to return a value of false, thereby causing the shipment not to be added. This check assumes that, at load execution, not all $q_i$ will be able to be transported and that a partial shipment is not desirable. Although there is a chance that there will be sufficient capacity available if some of the shipments currently in the load choose to be removed prior to scheduling, this possibility is not considered.

Second, since the final size of each shipment in the load is now known, the estimated sizes in (18) can be replaced with the actual size $q_j^{+i}$ of each of the other shipments in $L$, so that a value of false is returned if

$$TLC\left( c_i\left(q_i, \left[q_j^{+i}\right]_{j \in L \setminus i}\right), q_i \right) > TLC_i^0.$$
Conclusion

The major potential advantage of the proposed distributed approach is the limited amount of data that needs to be made public in order to be able to participate in the consolidation process. This can significantly expand the number of shipments available for consolidation, thereby increasing the scope of multi-firm, collaborative transport planning and deducing the need for third-party logistics providers.

Acknowledgements

The first author would like to thank Kai Furmans for his hospitality during the initial writing of this paper. This research is supported, in part, by the National Science Foundation under Grant CMS-0229720 (NSF/USDOT).

References