North Carolina State University is a land-grant university and a constituent institution of The University of North Carolina

Department of Mathematics

NC STATE UNIVERSITY

MA 305 Elem Linear Algebra, first mid-semester examination, February 10

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Your Name: ____________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 3 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **45 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **one** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later. You will have **75 minutes** to do this test.

Good luck!

Problem 1	____
2	____
3	____
Total	____

If you are taking the exam later, please sign the following statement:

I, ____________________*, affirm that I have no knowledge of the contents of this exam.*

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Signature

Problem 1 (20 points, 4 points each part)

(a) Please restate Fibonacci's famous rabbit problem.

(b) A square matrix is said to be *symmetric* if $A^T = A$. For an arbitrary matrix $B \in \mathbb{R}^{m \times n}$, is the product $B^T \cdot B$ then always a symmetric matrix? True or false? Please explain.

(c) In Maple, how does one compute the inverse of a matrix? How can one check the result to be the inverse matrix? Please give Maple commands.

(d) An algebraic group $(S; \text{mul}, \text{inv}, \text{id})$, where S is a set of elements, mul is the binary operator, id the identity element, and inv the inverse operator, is called *Abelian* if the binary operator satisfies the law of commutativity. Is the set of invertible square matrices of dimension n with real number entries and matrix multiplication as its binary operator an Abelian group? Please explain.

(e) When performing Gaussian elimination on a coefficient matrix A, it is sometimes useful not only to obtain the row echelon form U but also the transforming matrix T such that $T \cdot A = U$. Please give a scenario where T becomes useful.

Problem 2 (15 points, 5 points each part): Consider the following system of linear equations in the unknowns x, y, z and with parametric coefficients in a, b, c:

$$\begin{aligned} -2x + 3y - z &= -5, \\ 4x - 6y + 3z &= a, \\ 2x - 3y + bz &= c. \end{aligned}$$

(a) Please convert the augmented coefficient matrix of the above system to row echelon form.

$$\begin{bmatrix} -2 & 3 & -1 & -5 \\ 4 & -6 & 3 & a \\ 2 & -3 & b & c \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 3 & -1 & -5 \\ 0 & 0 & 1 & a-10 \\ 0 & 0 & b-1 & c-5 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 3 & -1 & -5 \\ 0 & 0 & 1 & a-10 \\ 0 & 0 & 0 & c-5-(b-1)(a-10) \end{bmatrix}.$$

(b) Find conditions for a, b, c (equations and/or inequalities) for which the above linear system is consistent.

(c) Assuming that a, b, c satisfy the conditions derived in (b), please give the complete solution of the above linear system.

Problem 3 (10 points, 5 points each part): Consider the following matrix product:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}}_{E_4} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}}_{E_3} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2} \cdot \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{E_1} \cdot \underbrace{\begin{bmatrix} 0 & 2 & -5 \\ 0 & -1 & 3 \\ 1 & 0 & 0 \end{bmatrix}}_{A}$$

Here the matrix A is multiplied from the left by four elementary matrices.

(a) Please describe in words the elementary row operation that each of E_1, E_2, E_3, E_4 represents.

(b) Please perform the four matrix products/row operations on A.