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Department of Mathematics

NC STATE UNIVERSITY

MA 305 Intro Elem Lin Algebra, first mid-semester examination, Feb 17, 2000

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Your Name: ____________________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 3 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **47 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	_____
2	_____
3	_____
Total	_____

If you are taking the exam later, please sign the following statement:

I, ______________________*, affirm that I have no knowledge of the contents of this exam.*

____
Signature

Problem 1 (20 points)

(a, 3 pts) Please name the countries of birth for the mathematicians C. F. Gauss, M.-E. C. Jordan, and Fibonacci.

Germany, France, Italy

(b, 5 pts) This is a variant of Fibonacci's famous rabbit problem: Suppose you start out with one newly born pair of rabbits, but each pair needs 2 months to reach maturity but thereafter gives birth to 2 pairs (2 male and 2 female) after every month. How many pair of rabbits are there after 10 months?

$f_{n+3} = f_{n+2} + 2f_n$.

month	0	1	2	3	4	5	6	7	8	9	10
#pairs	1	1	1	3	5	7	13	23	37	63	109

(c, 4 pts) True or false: for any matrices $A, B, C \in \mathbb{R}^{n \times n}$ we have $(A \cdot B \cdot C)^T = C^T \cdot B^T \cdot A^T$. Please explain.

True.

$(A \cdot B)^T = B^T \cdot A^T$ hence

$$(A \cdot B \cdot C)^T = ((A \cdot B) \cdot C)^T = C^T \cdot (A \cdot B)^T = C^T \cdot (B^T \cdot A^T) = C^T \cdot B^T \cdot A^T.$$

(d, 4 pts) In Maple, how does one compute the reduced row echelon form of a matrix? Please give Maple commands.

```
with(linalg); rref(A);
```

or

```
linalg[rref](A);
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(e, 4 pts) Please define the notion that a binary operation on a set is commutative. For the set of $n \times n$ matrices over the reals, is matrix addition commutative?

Let $\circ$ be the operation. $\forall a, b \in S \colon a \circ b = b \circ a$.

Yes, $+$ on $\mathbb{R}^{n \times n}$ is commutative.

Problem 2 (15 points): Consider the following augmented matrix of a system of linear equations, where the first 3 columns correspond to the variabes x, y, z and where a and b are real parameters.

$$\begin{bmatrix} 2 & 0 & 1 & \vdots & 1 \\ 0 & a & a & \vdots & a \\ 0 & b & b & \vdots & 0 \end{bmatrix} \tag{1}$$

(a, 5pts) For which values of a and b is the matrix (1) in row echelon form? Please give all the conditions.

$b = 0.$

(b, 5pts) Please perform the back-substitution for those values given in part (a).

Case $a = 0$: $2x + z = 1 \Longrightarrow x = 1/2 - 1/2z \quad z = z \quad y = y.$

Case $a \neq 0$: $ay + az = a \Longrightarrow y = 1 - z \quad z = z \quad x = 1/2 - 1/2z.$

(c, 5pts) Please solve the corresponding system for the values of a and b for which matrix (1) is not in row echelon form. Please also indicate for which values there is no solution.

$b \neq 0$:

Case $a = 0$: $\begin{bmatrix} 2 & 0 & 1 & \vdots & 1 \\ 0 & b & b & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix} \quad z = z \quad by + bz = 0 \Longrightarrow y = -z \quad x = 1/2 - 1/2z.$

Case $a \neq 0$: $\begin{bmatrix} 2 & 0 & 1 & \vdots & 1 \\ 0 & a & a & \vdots & a \\ 0 & 0 & 0 & \vdots & -b \end{bmatrix}$. No solution.

Problem 3 (12 points): Suppose you have a matrix $A \in \mathbb{R}^{4\times 4}$.

(a, 4 pts) Please write (explicitly in form of a matrix) an elementary matrix E that effects the subtraction of the double of each entry in row 1 from the corresponding entry in row 4 by performing the product $E \cdot A$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{bmatrix}$$

(b, 4 pts) What elementary operation is performed by the product $A \cdot E$, where E is as in part (a)?

Subtract 2 times column 4 from column 1.

(c, 4 pts) Please write E^{-1}, where E is as in part (a), in form of an elementary matrix, that is in the form $E_{\mathrm{III}}(\dots)$.

$E_{\mathrm{III}}(4,1,4,2)$