

1st

4

$$\left(\begin{array}{cccc|c} 1 & 3 & 3 & 2 & 1 \\ 2 & 6 & 9 & 5 & 5 \\ -1 & -3 & 3 & 0 & 5 \end{array} \right) \xrightarrow[\substack{-2R_1+R_2 \\ R_1+R_3}]{} \left(\begin{array}{cccc|c} 1 & 3 & 3 & 2 & 1 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 6 & 2 & 6 \end{array} \right)$$

$$\xrightarrow{-2R_2+R_3} \left(\begin{array}{cccc|c} \textcircled{1} & 3 & 3 & 2 & 1 \\ 0 & 0 & \textcircled{3} & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

↑ ↑ ↑ ↑
u v w y

3rd
The pivots are
1, 3. (Pivots are not zero)

The ^(dependent) basic variables correspond to columns with pivots. u, w
" free variables " " " without pivots v, y

Let $v=v$ and $y=y$ then write out the equations and solve for the ^(dependent) basic variables in terms of the free variables.

$$3w + y = 3 \rightarrow w = \frac{-1}{3}y + 1$$

$$u + 3v + 3w + 2y = 1 \rightarrow u = -2 - 3v - y$$

So the general solution is $\begin{pmatrix} -2 - 3v - y \\ v \\ 1 - \frac{1}{3}y \\ y \end{pmatrix} = \vec{x}$ OR equivalently

$$\vec{x}_{\text{general}} = \begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + v \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} -1 \\ 0 \\ -1/3 \\ 1 \end{pmatrix}$$