

2) Consider
$$\begin{aligned} 3x - y + 2z &= 1 \\ -x + 2y + 4z &= 3 \\ 2x + y + az &= b \end{aligned}$$

$$\left(\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ -1 & 2 & 4 & 3 \\ 2 & 1 & a & b \end{array} \right) \xrightarrow[\frac{2}{3}R_2 + R_3]{\frac{1}{3}R_1 + R_2} \left(\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 0 & 5/3 & 14/3 & 10/3 \\ 0 & 5/3 & a - \frac{4}{3} & b - \frac{2}{3} \end{array} \right) \xrightarrow[3 \cdot R_3]{3 \cdot R_2} \left(\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 0 & 5 & 14 & 10 \\ 0 & 5 & 3a-4 & 3b-2 \end{array} \right) \xrightarrow{-R_2 + R_3} \left(\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 0 & 5 & 14 & 10 \\ 0 & 0 & 3a-18 & 3b-12 \end{array} \right)$$

$$\xrightarrow{\frac{1}{3} \cdot R_3} \left(\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 0 & 5 & 14 & 10 \\ 0 & 0 & a-6 & b-4 \end{array} \right)$$

a) The system is inconsistent iff $a-6=0$ and $b-4 \neq 0 \Leftrightarrow a=6$ and $b \neq 4$.

That is to say when $a=6$ and $\forall b \in \mathbb{R} \setminus \{4\}$ the system is inconsistent.

b) There is a unique solution just in the case that $a-6 \neq 0$ and b any real

i.e. $\forall a \in \mathbb{R} \setminus \{6\}$ and $\forall b \in \mathbb{R}$

c) There are infinitely many solutions whenever $a-6=0$ and $b-4=0$ i.e. $a=6, b=4$