North Carolina State University is a land-grant university and a constituent institution of The University of North Carolina

Department of Mathematics

NC STATE UNIVERSITY

MA 305 Intro Elem Lin Algebra, first mid-semester examination, Feb 12, 2004
kaltofen@math.ncsu.edu (email)
www.math.ncsu.edu/~kaltofen/courses/LinAlgebra/Spring03/ (URL)

919.515.8785 (phone)
919.515.3798 (fax)

Your Name: ________________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 3 problems, which are subdivided into 12 questions, where each question counts for the explicitly given number of points, adding to a total of **43 points**. Please write your answers in the spaces indicated, or below the questions **using the back of the sheets for all scratch work**. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	____
2	____
3	____
Total	____

If you are taking the exam later, please sign the following statement:

I, ____________________, *affirm that I have no knowledge of the contents of this exam.*

____

Signature

Problem 1 (18 points)

(a, 3 pts) We know Carl Friedrich Gauß for his elimination algorithm and his work on magnetism. Do Gauß's and Benjamin Franklin's lifetimes overlap? Also, please name a third contribution to science by Gauß.

(b, 3 pts) True or false: any diagonal matrix is in reduced row echelon form. Please explain.

(c, 4 pts) Suppose that A, B are symmetric $m \times n$ matrices. Is it true that $A + 2B$ is a symmetric matrix? Explain your answer.

(d, 4 pts) Please show that $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \notin \mathrm{GL}_2(\mathbb{R})$ by proving that for any $B \in \mathbb{R}^{2\times 2}$ one has $A \cdot B \neq I_2$.

(e, 4 pts) Please compute the following matrix product: $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} u & 0 & 0 \\ v & w & 0 \\ r & s & t \end{bmatrix}$

Problem 2 (13 points): Consider the following augmented matrix of a system of three linear equations, where a and b are real parameters:

$$A = \begin{bmatrix} 1 & -1 & 2 & b & \vdots & 0 \\ a & 2 & 3a+2 & 0 & \vdots & a+2 \\ 0 & 0 & 0 & b & \vdots & 1 \end{bmatrix}$$

Here columns 1–4 correspond to the variables x, y, z, w, respectively.

(a, 3 pts) Please give the conditions for the parameters a, b such that (i) A is in row echelon form; (ii) the system is consistent.

(b, 3 pts) For the case that the system is consistent and $a = 0$, please determine all solutions.

(c, 3 pts) For the case that the system is consistent and $a = -2$, please determine all solutions.

(d, 4 pts) For the case that the system is consistent and $a \notin \{-2, 0\}$, please determine all solutions.

Problem 3 (12 points): For a real parameter $a \neq 0$ consider the following matrix product:

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_4} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/a & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_3} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{E_2} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}}_{E_1} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ a & 0 & 0 \end{bmatrix}}_{A}$$

Here the matrix A is multiplied from the left by four elementary matrices.

(a, 4 pts) Please write each elementary matrix E_1, E_2, E_3, E_4 in the form of $E_I(\ldots)$ or $E_{II}(\ldots)$ or $E_{III}(\ldots)$.

(b, 4 pts) Please write each inverse matrix $E_1^{-1}, E_2^{-1}, E_3^{-1}, E_4^{-1}$ both in the form of $E_I(\ldots)$ or $E_{II}(\ldots)$ or $E_{III}(\ldots)$ and as 3×3 matrices.

(c, 4 pts) Please compute $E_1^{-1} \cdot (E_2^{-1} \cdot (E_3^{-1} \cdot E_4^{-1}))$ from right to left by performing the elementary row operations given by the elementary matrices in (b). Please show all intermediate results.