

**NC STATE UNIVERSITY**

MA 305 Intro Elem Lin Algebra, first mid-semester examination, Feb 13, 2003  
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*Your Name:* SOLUTION

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 3 problems, which are subdivided into 13 questions, where each question counts for the explicitly given number of points, adding to a total of **46 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **one** 8.5in  $\times$  11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 \_\_\_\_\_

2 \_\_\_\_\_

3 \_\_\_\_\_

Total \_\_\_\_\_

If you are taking the exam later, please sign the following statement:

I, \_\_\_\_\_, *affirm that I have no knowledge of the contents of this exam.*

\_\_\_\_\_  
Signature

**Problem 1** (20 points)

- (a, 3 pts) Most CRT computer monitors have a button which will *degauss* the monitor. From where is the term *degauss* derived?

*The mathematician C. F. Gauss or a gauss which is a unit of magnetism*

- (b, 3 pts) Consider the classic fibonacci sequence  $f_{n+2} = f_{n+1} + f_n$  with  $f_0 = a$  and  $f_1 = b$ . If  $f_4 = 8$  and  $f_5 = 11$  find  $a$  and  $b$ .

$$a=7, b=-2$$

- (c, 3 pts) Suppose that in a Maple session you have performed the command package with(LinearAlgebra).

Please give all necessary commands to compute  $\left( \left( \begin{bmatrix} 1 & 3 & 5 \\ -5 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -3 & 1 \\ 1 & -4 \end{bmatrix} \right)^T \right)^{-1}$ .

$$A:=Matrix([[1,3,5],[-5,4,6]]); B:=<<2,-3,1>|<3,1,-4>>; \\ MatrixInverse(Transpose(A.B));$$

For parts (d)-(f) let  $S = \{D \in \mathbb{R}^{n \times n} \mid D \text{ is diagonal and } D_{i,i} \neq 0 \text{ for all } i \in \{1, 2, \dots, n\}\}$

- (d, 4 pts) For any matrix  $D \in S$  show that  $D^{-1} \in S$  and show how to compute  $D^{-1}$ .

$D^{-1}$  is a matrix defined by  $D_{i,j}^{-1} = 0$  if  $i \neq j$  and  $D_{i,i}^{-1} = \frac{1}{D_{i,i}}$  for all  $i$ . So  $D^{-1}$  is diagonal and since  $D_{i,i} \neq 0$  then  $D_{i,i}^{-1} \neq 0$  for all  $i$ . Thus  $D^{-1} \in S$ .

- (e, 4 pts) True or false. If  $D, E \in S$  then  $D \cdot E \in S$ ? Please give a brief explanation.

*True. Since  $D, E$  are diagonal then  $D \cdot E$  is diagonal. Also  $(D \cdot E)_{i,i} = D_{i,i}E_{i,i} \neq 0$  since  $D_{i,i} \neq 0$  and  $E_{i,i} \neq 0$  for all  $i$ . Thus  $D \cdot E \in S$ .*

- (f, 3 pts) True or false. For any  $D, E, F \in S$ :  $(D \cdot E) \cdot F = D \cdot (E \cdot F)$ . Please explain.

*True. Matrix multiplication is associative.*

**Problem 2** (14 points): Consider the following system of linear equations, where  $a$  and  $b$  are real parameters.

$$\begin{aligned} x + 3y + 3z &= 6 \\ x + 4y + 7z &= 5 \\ y + a^2z &= b \end{aligned}$$

(a, 3 pts) Write the augmented coefficient matrix of the system and write it in row echelon form.

$$\left[ \begin{array}{ccc|c} 1 & 3 & 3 & 6 \\ 1 & 4 & 7 & 5 \\ 0 & 1 & a^2 & b \end{array} \right] \Rightarrow \left[ \begin{array}{ccc|c} 1 & 3 & 3 & 6 \\ 0 & 1 & 4 & -1 \\ 0 & 0 & a^2 - 4 & b + 1 \end{array} \right]$$

(b, 3 pts) For which values of  $a$  and  $b$  is the system inconsistent? Please give all possible values.

$$a = \pm 2 \text{ and } b \neq -1$$

(c, 4 pts) For which values of  $a$  and  $b$  does the system have a unique solution? For each case please compute the solution in terms of  $a$  and  $b$ .

$$a \neq \pm 2$$

$$z = \frac{b+1}{a^2-4}$$

$$y = -1 - 4 \left( \frac{b+1}{a^2-4} \right)$$

$$x = 9 + 9 \left( \frac{b+1}{a^2-4} \right)$$

(d, 4 pts) For which values of  $a$  and  $b$  does the system have infinitely many solutions? For each case please compute the solution in terms of  $a$  and  $b$ .

$$a = \pm 2 \text{ and } b = -1$$

$$z = z$$

$$y = -1 - 4z$$

$$x = 9 + 9z$$

**Problem 3** (12 points): Let  $A \in \mathbb{R}^{3 \times 3}$  and suppose the following is true.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}_{E_4} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_3} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{E_2} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1} A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In this problem you are to compute  $A$  from the elementary matrices  $E_1, E_2, E_3, E_4$ . Please perform the following steps.

(a, 4 pts) Write each elementary matrix in the form of  $E_I(\dots)$  or  $E_{II}(\dots)$  or  $E_{III}(\dots)$ .

$$\begin{aligned} E_1 &= E_{III}(3, 1, 2, 3) \text{ or } E_{III}(1, 2, 3) \\ E_2 &= E_I(3, 2, 3) \text{ or } E_I(2, 3) \text{ or } E_I(3, 2) \\ E_3 &= E_{II}(3, 2, 1/2) \text{ or } E_{II}(2, 1/2) \\ E_4 &= E_{III}(3, 3, 2, -2) \text{ or } E_{III}(3, 2, -2) \end{aligned}$$

(b, 4 pts) For each elementary matrix in (a), please give its inverse as another elementary matrix in the form of  $E_I(\dots)$  or  $E_{II}(\dots)$  or  $E_{III}(\dots)$ . Also write each inverse as a  $3 \times 3$  matrix.

$$\begin{aligned} E_1^{-1} &= E_{III}(3, 1, 2, -3) \text{ or } E_{III}(1, 2, -3) \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ E_2^{-1} &= E_I(3, 2, 3) \text{ or } E_I(2, 3) \text{ or } E_I(3, 2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad E_3^{-1} = E_{II}(3, 2, 2) \text{ or } E_{II}(2, 2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ E_4^{-1} &= E_{III}(3, 3, 2, 2) \text{ or } E_{III}(3, 2, 2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

(c, 4 pts) Compute  $A$  from the elementary matrices in (b). (Hint, instead of doing matrix multiplication, use row operations instead.)

$$\begin{aligned} A &= E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} \\ E_3^{-1} E_4^{-1} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix} \\ E_2^{-1} E_3^{-1} E_4^{-1} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 4 \end{bmatrix} \\ A &= E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 0 & 1 \\ 0 & 2 & 4 \end{bmatrix} \end{aligned}$$