Your Name: SOLUTION
For purpose of anonymous grading, please do not write your name on the subsequent pages.

This examination consists of 3 problems, which are subdivided into 13 questions, where each question counts for the explicitly given number of points, adding to a total of 46 points. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult one 8.5in × 11in sheet with notes, but not your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have 75 minutes to do this test.

Good luck!

Problem 1  ____

2  ____

3  ____

Total  ____

If you are taking the exam later, please sign the following statement:

I, ____________________, affirm that I have no knowledge of the contents of this exam.

__________________________
Signature
Problem 1 (20 points)

(a, 3 pts) Most CRT computer monitors have a button which will *degauss* the monitor. From where is the term *degauss* derived?

*The mathematician C. F. Gauss or a gauss which is a unit of magnetism*

(b, 3 pts) Consider the classic fibonacci sequence \( f_{n+2} = f_{n+1} + f_n \) with \( f_0 = a \) and \( f_1 = b \). If \( f_4 = 8 \) and \( f_5 = 11 \) nd \( a \) and \( b \).

\[
a=7, \ b=-2
\]

(c, 3 pts) Suppose that in a Maple session you have performed the command package `with(LinearAlgebra)`.

Please give all necessary commands to compute

\[
A:=\text{Matrix}([[1,3,5],[1,4,6],[2,3,1]]);
B:=\langle\langle 2, -3, 1 \rangle | \langle 3, 1, -4 \rangle \rangle;
\text{MatrixInverse(Transpose(A.B))};
\]

For parts (d)-(f) let \( S = \{ D \in \mathbb{R}^{n \times n} \mid D \text{ is diagonal and } D_{i,i} \neq 0 \text{ for all } i \in \{1, 2, \ldots, n\} \} \)

(d, 4 pts) For any matrix \( D \in S \) show that \( D^{-1} \in S \) and show how to compute \( D^{-1} \).

\[
D^{-1} \text{ is a matrix defined be } D^{-1}_{i,j} = 0 \text{ if } i \neq j \text{ and } D^{-1}_{i,i} = \frac{1}{D_{i,i}} \text{ for all } i. \text{ So } D^{-1} \text{ is diagonal and since } D_{i,i} \neq 0 \text{ then } D^{-1}_{i,i} \neq 0 \text{ for all } i. \text{ Thus } D^{-1} \in S.
\]

(e, 4 pts) True or false. If \( D, E \in S \) then \( D \cdot E \in S \)? Please give a brief explanation.

*True. Since \( D, E \) are diagonal then \( D \cdot E \) is diagonal. Also \( (D \cdot E)_{i,i} = D_{i,i}E_{i,i} \neq 0 \text{ since } D_{i,i} \neq 0 \text{ and } E_{i,i} \neq 0 \text{ for all } i. \text{ Thus } D \cdot E \in S.\)*

(f, 3 pts) True or false. For any \( D, E, F \in S \): \( (D \cdot E) \cdot F = D \cdot (E \cdot F) \). Please explain.

*True. Matrix multiplication is associative.*
Problem 2 (14 points): Consider the following system of linear equations, where $a$ and $b$ are real parameters.

\[
\begin{align*}
  x + 3y + 3z &= 6 \\
  x + 4y + 7z &= 5 \\
  y + a^2z &= b
\end{align*}
\]

(a, 3 pts) Write the augmented coefficient matrix of the system and write it in row echelon form.

\[
\begin{bmatrix}
  1 & 3 & 3 & \vdots & 6 \\
  1 & 4 & 7 & \vdots & 5 \\
  0 & 1 & a^2 & \vdots & b \\
\end{bmatrix} \Rightarrow \begin{bmatrix}
  1 & 3 & 3 & \vdots & 6 \\
  0 & 1 & 4 & \vdots & -1 \\
  0 & 0 & a^2 - 4 & \vdots & b + 1 \\
\end{bmatrix}
\]

(b, 3 pts) For which values of $a$ and $b$ is the system inconsistent? Please give all possible values.

\[
a = \pm 2 \text{ and } b \neq -1
\]

(c, 4 pts) For which values of $a$ and $b$ does the system have a unique solution? For each case please compute the solution in terms of $a$ and $b$.

\[
\begin{align*}
  a &\neq \pm 2 \\
  z &= \frac{b + 1}{a^2 - 4} \\
  y &= -1 - 4 \left( \frac{b + 1}{a^2 - 4} \right) \\
  x &= 9 + 9 \left( \frac{b + 1}{a^2 - 4} \right)
\end{align*}
\]

(d, 4 pts) For which values of $a$ and $b$ does the system have infinitely many solutions? For each case please compute the solution in terms of $a$ and $b$.

\[
\begin{align*}
  a &= \pm 2 \text{ and } b=-1 \\
  z &= z \\
  y &= -1 - 4z \\
  x &= 9 + 9z
\end{align*}
\]
Problem 3 (12 points): Let $A \in \mathbb{R}^{3 \times 3}$ and suppose the following is true.

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1/2 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

In this problem you are to compute $A$ from the elementary matrices $E_1, E_2, E_3, E_4$. Please perform the following steps.

(a, 4 pts) Write each elementary matrix in the form of $E_{III}(\ldots)$ or $E_{II}(\ldots)$ or $E_{I}(\ldots)$.

\[
E_1 = E_{III}(3,1,2,3) or E_{III}(1,2,3) \\
E_2 = E_{I}(3,2,3) or E_{I}(2,3) or E_{I}(3,2) \\
E_3 = E_{II}(3,2,1/2) or E_{II}(2,1/2) \\
E_4 = E_{III}(3,3,2,-2) or E_{III}(3,2,-2)
\]

(b, 4 pts) For each elementary matrix in (a), please give its inverse as another elementary matrix in the form of $E_{III}(\ldots)$ or $E_{II}(\ldots)$ or $E_{I}(\ldots)$. Also write each inverse as a $3 \times 3$ matrix.

\[
E_1^{-1} = E_{III}(3,1,2,-3) or E_{III}(1,2,-3) \\
E_2^{-1} = E_{I}(3,2,3) or E_{I}(2,3) or E_{I}(3,2) \\
E_3^{-1} = E_{II}(3,2,2) or E_{II}(2,2) \\
E_4^{-1} = E_{III}(3,3,2,2) or E_{III}(3,2,2)
\]

(c, 4 pts) Compute $A$ from the elementary matrices in (b). (Hint, instead of doing matrix multiplication, use row operations instead.)

\[
A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}
\]