North Carolina State University is a land-grant university and a constituent institution of The University of North Carolina

Department of Mathematics

NC STATE UNIVERSITY

MA 305 Intro Elem Lin Algebra, second mid-semester examination, Mar 26, 2002

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Your Name: ________________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 3 problems, which are subdivided into 9 questions, where each question counts for the explicitly given number of points, adding to a total of **42 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **two** 8.5in $\times$ 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

Total _____

If you are taking the exam later, please sign the following statement:

I, ____________________, *affirm that I have no knowledge of the contents of this exam.*

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Signature

Problem 1 (20 points)

(a, 4 pts) True or false: a matrix $A \in \mathbb{R}^{n\times n}$ in which two different rows are the same, i.e., for which the entries in matching columns are equal, is singular. Please explain.

(b, 4 pts) Suppose that you have computed the row echelon form U of a matrix $A \in \mathbb{R}^{n\times n}$ and the product T of elementary matrices that transforms A into U such that $T \cdot A = U$. Furthermore, assume that T is lower triangular. Is there an easy way to compute the determinant of A from T and U? If so, how would you do it?

(c, 4 pts) Consider $\mathbb{R}^2$ with $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \oplus \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}$ for addition and $\alpha \odot \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \alpha x_1 \\ y_1/\alpha \end{bmatrix}$ for all $\alpha \neq 0$ and $0 \odot \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ for scalar multiplication. Is $\mathbb{R}^2$ with these operations a vector space? Please justify your answer.

(d, 4 pts) Consider the following subset $S \subset \mathbb{R}^{2\times 2}$: $S = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid \det(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = 1\})$. With the usual matrix addition and multiplication of matrices by scalars, does S form a subspace of $\mathbb{R}^{2\times 2}$? Please justify your answer.

(e, 4 pts) In $\mathbb{R}^3$, consider an infinite plane that contains the origin. This plane is the solution set to a system of linear equations (with the coordinates x, y, z as unknowns). How many equations in the minimum are necessary for obtaining the full plane as solution? Please explain.

Problem 2 (10 points): Consider the following linear system given as a matrix equation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ s & 0 & 3 & 0 \\ 0 & t & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}. \tag{1}$$

Here a, b, c, d, s, t are parameters.

(a, 4pts) By Cramer's rule, please write down two matrices so that the solution x_4 of (1) is the quotient of their determinants.

(b, 6pts) Please compute the values of these two matrix determinants by minor (also known as cofactor) expansion. From them, determine the value of x_4 (as a function in a, b, c, d, s, t).

Problem 3 (12 points): Consider the following vectors in $\mathbb{R}^4$:

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 2 \\ -4 \\ 4 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ -3 \\ 2 \\ -6 \end{bmatrix}, v_4 = \begin{bmatrix} 3 \\ -1 \\ 6 \\ -2 \end{bmatrix}.$$

(a, 6 pts) Please pare down the list v_1, v_2, v_3, v_4 to form a basis of $\text{Span}_{\mathbb{R}}(v_1, v_2, v_3, v_4)$.

(b, 6 pts) Please compute a matrix $A \in \mathbb{R}^{2\times 4}$ such that $\text{Span}_{\mathbb{R}}(v_1, v_2, v_3, v_4)$ is the right null space of A.