

NC STATE UNIVERSITY

MA 305 Intro Elem Lin Algebra, final examination, May 10, 2001
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Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **48 points**. Please write your answers in the spaces indicated, or below the questions, using the back of the sheets for scratch work and for completing the answers, if necessary. You are allowed to consult **three** 8.5in \times 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **120 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

5 _____

Total _____

If you are taking the exam later, please sign the following statement:

I, _____, *affirm that I have no knowledge of the contents of this exam.*

Signature

Problem 1 (18 points)

(a, 4 pts) Please name two separate problems in linear algebra whose solution requires the computation of a null space for a matrix.

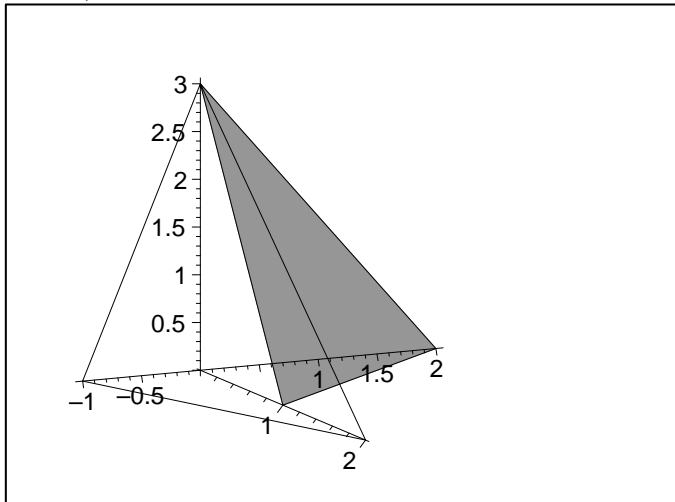
(b, 4 pts) Consider $v = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$. Please compute the infinity norm of v , namely $\|v\|_\infty$, and the 1-norm (Manhattan norm) of v , namely $\|v\|_1$.

(c, 4 pts) Consider the following function on $\mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}$: $\left\langle \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \right\rangle = x_1y_2 + x_2y_1$. Is this function an inner product in the vector space \mathbb{R}^2 ? Please explain.

(d, 2 pts) Suppose that V is a vector space over \mathbb{R} and $N: V \rightarrow \mathbb{R}_{\geq 0}$ is a norm function. Please state the triangle inequality for the function N .

(e, 4 pts) Consider the following function on $\mathbb{R}^2 \rightarrow \mathbb{R}^2$: $F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$. Is F a linear transformation on \mathbb{R}^2 ? Please explain.

Problem 2: (5 pts) Consider the 3-dimensional rotation around the z-axis (parallel to the x-y plane) by 270 degrees counter-clockwise.



In the above plot, the 3-dimensional triangle with vertices at $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$ is shown both in original (the shaded) and in rotated (the wireframe) position. Please give the 3×3 matrix which when multiplied by any vector in \mathbb{R}^3 effects this rotation.

Problem 3: (6 pts) A computer program takes as input a 2-dimensional array with m rows and n columns. For selected values of m and n , the following running times (in milliseconds) were observed:

m	10	10	10	15	15	20	20
n	5	10	15	10	20	15	20
Time(m, n)	0.4	0.6	0.8	1.3	2.7	3.1	5.0

Analysis of the program suggests that the running time is estimated as

$$c_1 \cdot m^2 + c_2 \cdot n^2 + c_3 \cdot m + c_4 \cdot n + c_5 \tag{1}$$

for some constants c_1, c_2, c_3, c_4, c_5 . Please write down a 7×5 matrix A and a 7-dimensional

vector b such that the solution to the corresponding least squares problem $A \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} \approx b$ yields

a best fit to the model (1). You **do not** need to compute the solution.

Problem 4: (10 pts) Consider the basis

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

for a subspace of \mathbb{R}^4 .

(a, 5 pts) Please compute the Gram-Schmidt orthogonalization (as done in class, **without normalizing** the vectors to unit length) of this basis.

(b, 5 pts) By use of the results of part a, please compute the orthogonal projection of the vector

$$w = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ onto } \text{Span}_{\mathbb{R}}(v_1, v_2, v_3).$$

Problem 5: (9 pts)

- (a, 6 pts) Consider the 2×2 matrix $A = \begin{bmatrix} 3 & 12 \\ -2 & -7 \end{bmatrix}$. **Without** the use of a **computer or a calculator**, please compute the eigenvalues and corresponding eigenvectors for A . Please show your work.

- (b, 3 pts) Why are eigenvalues/vectors important? Please give a reason by stating an important mathematical problem that hinges on them.