North Carolina State University
Department of Mathematics
College of Physical and Mathematical Sciences

MA 305 Elem Linear Algebra
First mid-semester examination
September 18, 1997

Erich Kaltofen
Professor

Your Name: ______________________
For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 3 questions, each question counting for the given number of points, adding to a total of 20 points. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult a single $8.5' \times 11'$ sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have 75 minutes to do this test.

Good luck!

Problem 1	____
2	____
3	____
Total	____

If you are taking the exam later, please sign the following statement:

I, ________________ *affirm that I have not contacted my class mates about the contents of this exam.*

____
Signature

Problem 1 (9 points, 1.5 points for each part): Please answer the following questions.

(a) Please explain the difference between the Gaussian elimination process and the Gauss-Jordan elimination process.

(b) Please give two matrices $A, B \in \mathbb{R}^{2\times 2}$ such that

$$(A+B)(A-B) \neq A^2 - B^2.$$

(c) Consider the Fibonacci recursion $f_{i+2} = f_{i+1} + f_i$. What initial values $x = f_0$ and $y = f_1$ yield $f_6 = f_7 = -1$?

(d) Suppose you have a set S and a binary operator $\bullet$ that maps a pair $(a,b) \in S \times S$ to $(a \bullet b) \in S$. An element $u \in S$ is a unit (also called zero or identity) element with respect to $\bullet$ when what property is satisfied?

(e) Suppose $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$. If A is a non-singular matrix, then the linear system $Ax = b$ has a single, unique solution. Please explain why.

(f) A linear system is *overdetermined* if there are more equations than variables. Suppose that an overdetermined linear system is consistent. Is it then possible that such a system has a single, unique solution? Please explain.

Problem 2 (5 points, 2.5 points for each part): Consider the following system of linear equations:

$$\begin{aligned} 2x + 8y + 4z &= 7 \\ -4x - 16y - 10z &= -9 \\ -4z &= 10 \end{aligned}$$

(a) Please convert the augmented coefficient matrix of the above system to row echelon form by elementary row operations.

(b) Please give the complete solution of the above linear system.

Problem 3 (6 points, 1.5 points for each part): Suppose you have a matrix $A \in \mathbb{R}^{4\times 4}$.

(a) Please write (explicitly in form of a matrix) an elementary matrix E_1 that effects the exchange of row 2 and row 4 in A by performing the product $E_1 \cdot A$.

(b) Please write (explicitly in form of a matrix) an elementary matrix E_2 that effects the subtraction of each entry in column 2 from the corresponding entry in column 4 in A by performing the product $A \cdot E_2$.

(c) Please write (explicity in form of a matrix) the matrix $(E_1 \cdot E_2)^{-1}$.

(d) Please give a Maple command (possible using procedures provided in my `refpkg` package) that computes the solution of problem (c).