

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, first mid-semester examination, Sep 21, 1999
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Your Name: SOLUTION

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 4 problems, which are subdivided into 12 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **one** 8.5in \times 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

Total _____

Problem 1 (12 points, 4 points each part)

- (a) Please restate Fibonacci's famous rabbit problem.

In a courtyard enclosed by a wall, a newly born pair of rabbits, one male and one female, is placed. The pair takes one month to mature, and after that they give birth after every month to a new pair of rabbits, exactly one male and one female. These rabbits have the same maturation and gestation periods. How many pairs of rabbits are in the courtyard after 10 months?

- (b) Please explain the difference between a prescriptive and a descriptive mathematical model.

***Prescriptive:** the model is used to mimic a possibly realistic situation. An example is the Fibonacci rabbit model, and another is a fractal landscape. A modern word for it is "virtual reality."*

***Descriptive:** the model is used to approximate a natural process, the so-called "real world." An example is a model for hurricane forecasting.*

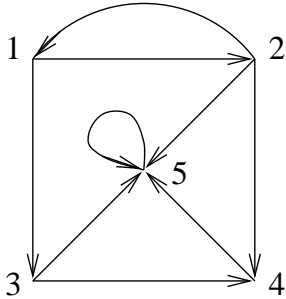
- (c) The mathematical notion of a graph is a set-theoretic object. (Note: graph vs. **digraph**). Please give the definition.

A graph is a pair of sets, (V, E) . V is a finite set, the set of vertices, and E is a set of two element sets $\{a, b\}$ where both a and b are elements of V . The elements of E are called the edges.

Problem 2 (14 points): Consider the following digraph:

$$D = (\{1, 2, 3, 4, 5\}, \{(1, 2), (1, 3), (2, 1), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5), (5, 5)\}).$$

(a, 3pts) Please draw a picture of D .



(b, 3pts) Please write down the adjacency matrix M for D under the vertex order $(1, 2, 3, 4, 5)$.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

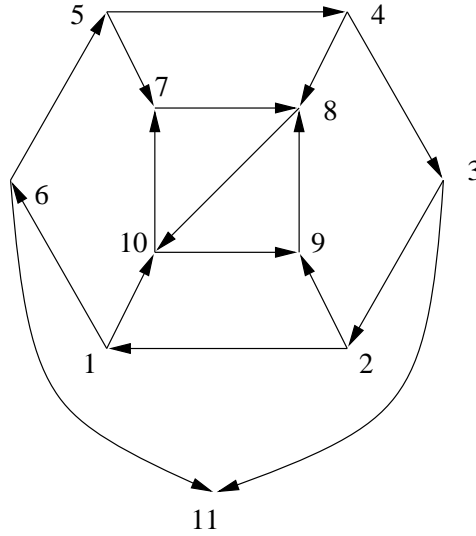
(c, 4pts) Please write down M^2 .

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(d, 4pts) Please write down the reachability matrix R for D under the vertex order $(1, 2, 3, 4, 5)$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3 (14 points):
Consider the following digraph:



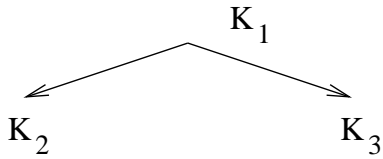
(a, 4pts) Please list the strong components of the above digraph.

$$K_1 = \{1, 2, 3, 4, 5, 6\}$$

$$K_2 = \{7, 8, 9, 10\}$$

$$K_3 = \{11\}$$

(b, 4pts) Please draw the digraph that is the condensation of the above digraph.



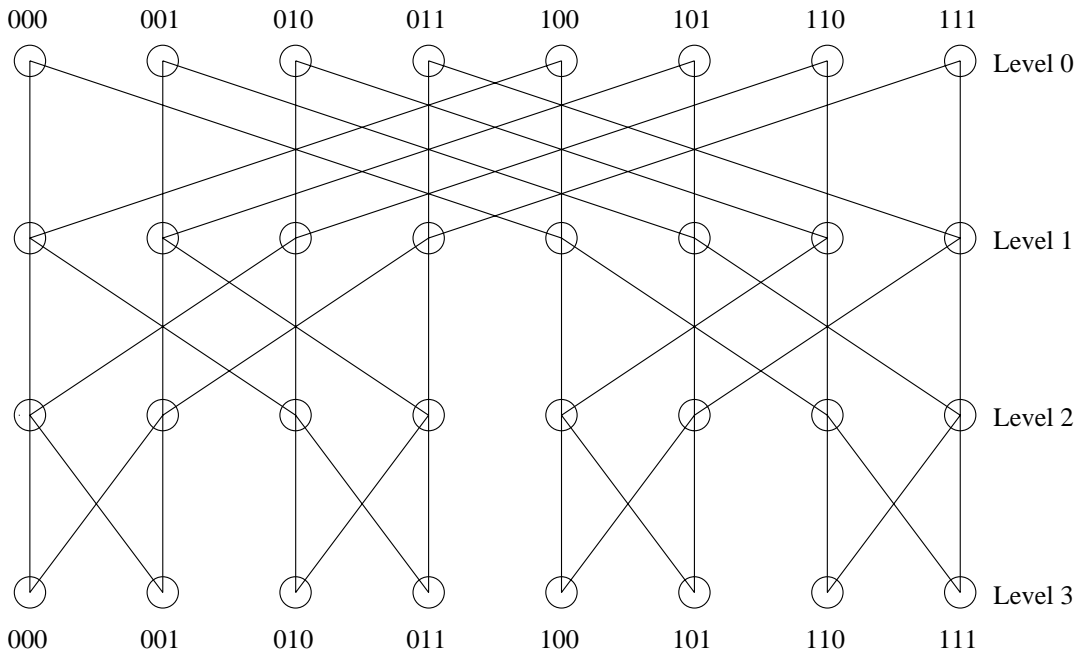
(c, 4pts) Please list a vertex basis for the condensation and from it derive a vertex basis for the above digraph.

$$B^* = \{K_1\}, \quad B = \{1\}$$

(d, 2pts) How many vertex bases does the above digraph have?

six

Problem 4 (10 points): Consider the butterfly graph with 8 nodes per level:



Each node is labelled $v_{\ell,abc}$, where ℓ is the horizontal level and abc is the binary vertical number in the picture.

Please list all distinct **shortest** paths (called “chains” by the textbook in the case of graphs) as a sequence of nodes between $v_{0,010}$ and $v_{0,100}$.

- $(v_{0,010}, v_{1,\boxed{1}10}, v_{2,\boxed{0}0}, v_{1,100}, v_{0,100})$
- $(v_{0,010}, v_{1,\boxed{1}10}, v_{2,110}, v_{1,\boxed{0}0}, v_{0,100})$
- $(v_{0,010}, v_{1,010}, v_{2,\boxed{0}0}, v_{1,000}, v_{0,\boxed{1}00})$
- $(v_{0,010}, v_{1,010}, v_{2,010}, v_{1,\boxed{0}0}, v_{0,\boxed{1}00})$

Note: the boxed subscripts indicate when the bits are changed to fit the target subscripts. The first bit can be changed either by the first edge of the path, or by the last. The second bit can be changed either by the second edge, or by the third edge.