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Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, first mid-semester examination, Sep 26, 2006

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Your Name: ____________________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 4 problems, which are subdivided into 16 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	_____
2	_____
3	_____
4	_____
Total	_____

Problem 1 (12 points)

(a, 8pts) Consider the following variant of Fibonnaci's problem. After taking one month to mature and one month to gestate, each pair of rabbits produces 2 pairs of newly born rabbits. Again, there is one newly born pair of rabbits placed in the courtyard at month 0.

i. Please write the recursive equation for f'_n, which denotes the number of pairs of rabbits at month n in the above variant, and give the values for $f'_0, \ldots, f'_4$.

ii. Please compute a closed form solution (in the format like the one given in class for the Fibonacci numbers) for f'_n.

(b, 4pts) For the following mathematical objects, please indicate whether the object is a **di**graph, or a graph, or both or neither. Please explain.

i. $(\{A,B\},\{(1,1),(2,2)\})$

ii. $(\{A,B\},\{\{A\},\{A,B\}\})$

iii. $(\{A,B\},\varnothing)$, where $\varnothing$ denotes the empty set.

iv. $\{\{A,B\},\{(B,A)\}\}$

Problem 2 (14 points): Consider the following digraph:

$$D = (\{1,2,3,4,5\}, \{(1,4),(2,2),(2,3),(2,4),(3,3),(3,4),(4,3),(4,5),(5,1)\}).$$

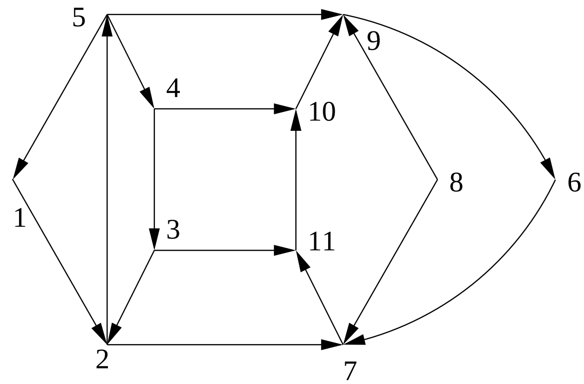
(a, 3pts) Please draw a picture of D.

(b, 3pts) Please write down the adjacency matrix M for D under the vertex order $(1,2,3,4,5)$.

(c, 4pts) Please write down M^2.

(d, 4pts) Please write down the reachability matrix R for D under the vertex order $(1,2,3,4,5)$.

Problem 3 (14 points):
Consider the following digraph:



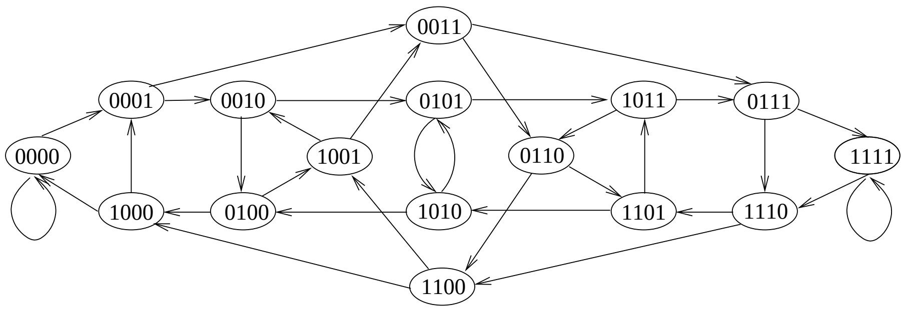
(a, 4pts) Please list the strong components of the above digraph.

(b, 4pts) Please draw the digraph that is the condensation of the above digraph.

(c, 4pts) Please list a vertex basis for the condensation and from it derive a vertex basis for the above digraph.

(d, 2pts) How many vertex bases does the above digraph have?

Problem 4 (10 points): Consider the n-dimensional de Bruijn digraph with 2^n vertices and $2^{n+1}-2$ arcs. Like in the hypercube, the vertices are n-bit numbers. There is an arc from each vertex denoted by $b_1 b_2 \ldots b_n$, where $b_j \in \{0,1\}$ for all j with $1 \leq j \leq n$, to the vertex denoted by $b_2 b_3 \ldots b_n 0$ and an arc to the vertex denoted by $b_2 b_3 \ldots b_n 1$. Below is a picture for $n = 4$. For example, there is an arc $(1\underline{001}, \underline{001}\mathbf{0})$ and an arc $(1\underline{001}, \underline{001}\mathbf{1})$.



(a, 2pts) For $n = 4$ (see the above picture) what is the distance from vertex 0101 to vertex 0000?

(b, 5pts) Please prove that for $n = 4$ the digraph has diameter no larger than 4 that is, the maximum distance between any two vertices is ≤ 4.

(c, 3pts) The *indegree* of a vertex is the maximum number of arcs "pointing at it." For arbitrary n, what is the maximal indegree among all of the vertices? Please explain.