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Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, second mid-semester examination, Nov 6, 2003

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Your Name: SOLUTION
For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 10 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). **Please do not use your own scratch paper.** You are allowed to consult **two** 8.5in × 11in sheets with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

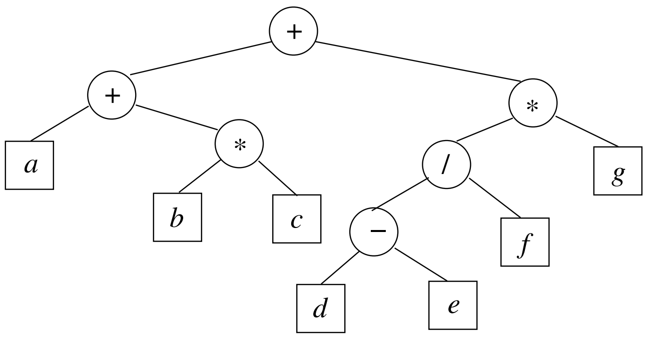
Good luck!

Problem 1	____
2	____
3	____
4	____
5	____
Total	____

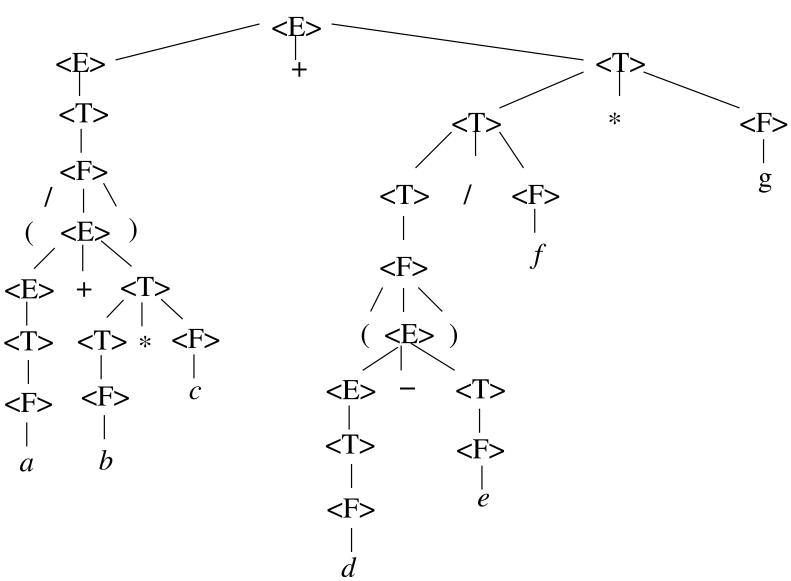
Problem 1 (14 points) Consider the following mathematical formula:

$$(a+b*c)+(d-e)/f*g \tag{1}$$

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precedence rules and left-to-right tie-breaking for operators of equal precedence.



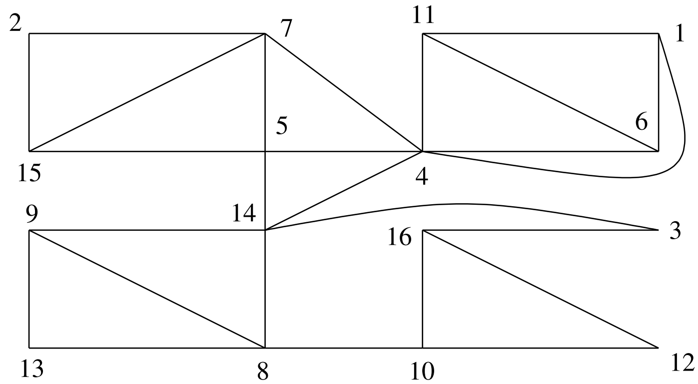
(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.



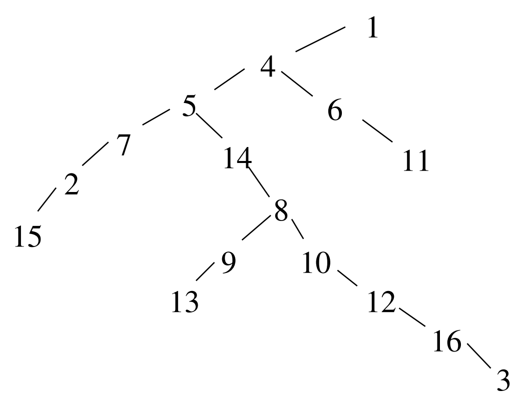
(c, 4pts) Please give **both** a prefix **and** postfix string of operators and variables, but with no parentheses, that represents the tree given under part (a).

Pre: $++a*bc*/-defg$.
Post: $abc*+de-f/g*+$.

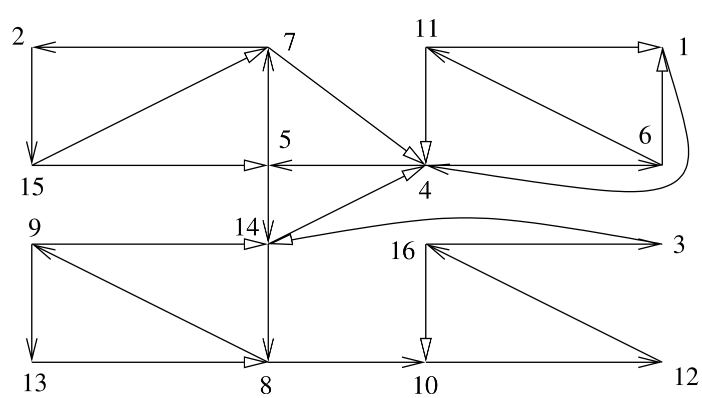
Problem 2 (10 points): Consider the following graph:



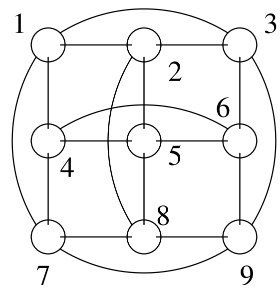
(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex **in numerical order**, starting at vertex 1.



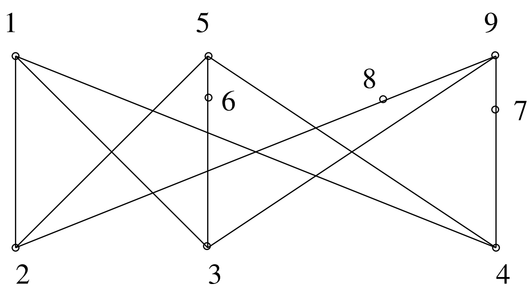
(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., please orient the edges so that the resulting digraph is strongly connected.



Problem 3 (12 points): Consider the 3×3 toric mesh (with the given vertex labeling):

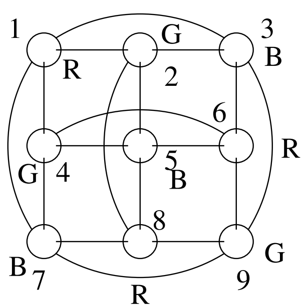


(a, 6pts) Please draw a subgraph that is homeomorphic to $K_{3,3}$. [Hint: choose as the first subset $\{1,5,9\}$ and as the second $\{2,3\}$ and another vertex.]



(b, 4pts) What is the chromatic number of the above 3×3 toric mesh? Please justify your answer.

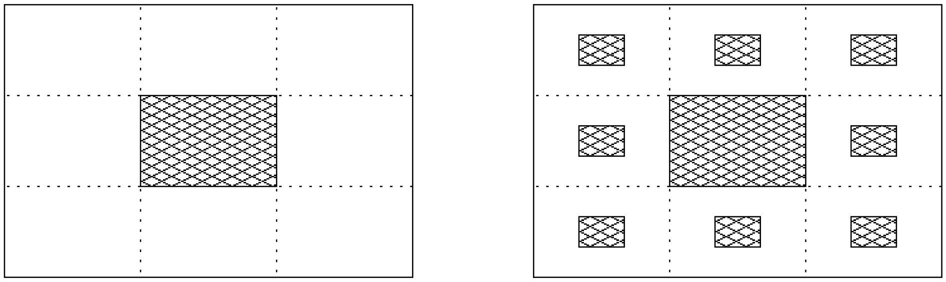
$\chi = 3$ *because there exists a 3-clique, namely the vertices 1, 2, 3, and 3-coloring is possible:*



(c, 2pts) Please give an example of a graph such that the maximum of the degrees of all the vertices is equal to the chromatic number minus 1.

$G = K_1 = \{\{1\}, \varnothing\}$*:* $\Delta = 0$, $\chi = 1$.
Or $G = K_2 = \{\{1,2\}, \{\{1,2\}\}\}$*:* $\Delta = 1$, $\chi = 2$.

Problem 4 (10 points): Please consider the following 2-dimensional sponge fractal:



Here you start with a rectangle of area A. You remove a concruent rectangle of area $1/9 \cdot A$ from the middle, and proceed recursively for all the remaining 8 concruent rectangles surrounding the "hole."

(a, 5pts) If the process of cutting out holes is continued to infinity, what is the remaining white area of the sponge fractal in terms of A. Please show your computation.

$$\textit{Area of holes} = A \cdot \left(\frac{1}{9} + \frac{8}{81} + \frac{64}{729} + \cdots + \frac{8^i}{9^{i+1}} + \cdots \right) = \frac{A}{9} \cdot \sum_{i=0}^{\infty} \left(\frac{8}{9} \right)^i = \frac{A}{9} \cdot \frac{1}{1-8/9} = A.$$

Remaining area: $A - A = 0$.

(b, 5pts) What is the length of boundary of the sponge, accounting for both the outside boundary and the boundaries of all the holes? Please show your computation.

$$\textit{Boundary of holes: } L \cdot \left(\frac{1}{3} + \frac{8}{9} + \cdots + \frac{8^i}{3^{i+1}} + \cdots \right) = \frac{L}{3} \cdot \sum_{i=0}^{\infty} \left(\frac{8}{3} \right)^i = \infty.$$

Problem 5 (4 points): Please give the definition of the Julia set for the iterating function $z^2 + 2$.

$$J_2 = \{ b \in \mathbb{C} \mid \exists B \in \mathbb{R}_{>0} \colon \forall i \geq 1 \colon z_i = z_{i-1}^2 + 2, z_0 = b \implies |z_i| \leq B \}.$$