

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, first mid-semester examination, Sep 25, 2003
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Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 4 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions, using the back of the sheets for scratch work and for completing the answers, if necessary. You are allowed to consult **one** 8.5in × 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

Total _____

Problem 1 (12 points)

- (a, 8pts) Consider the linear recursion $h_{n+2} = 2h_{n+1} + h_n$ for $n \geq 0$ with $h_0 = 1, h_1 = 1$. Please list the first h_0, \dots, h_8 . Please compute a closed form solution (in the format like the one given in class for the Fibonacci numbers) for h_n .

- (b, 4pts) Please give a descriptive model different from the famous models of the solar system, and a prescriptive model other than Fibonacci's rabbit problem.

Problem 2 (14 points): Consider the following digraph:

$$D = (\{1, 2, 3, 4, 5, 6\}, \{(1, 1), (2, 1), (2, 3), (2, 5), (3, 1), (3, 6), (4, 4), (5, 1), (5, 6)\}).$$

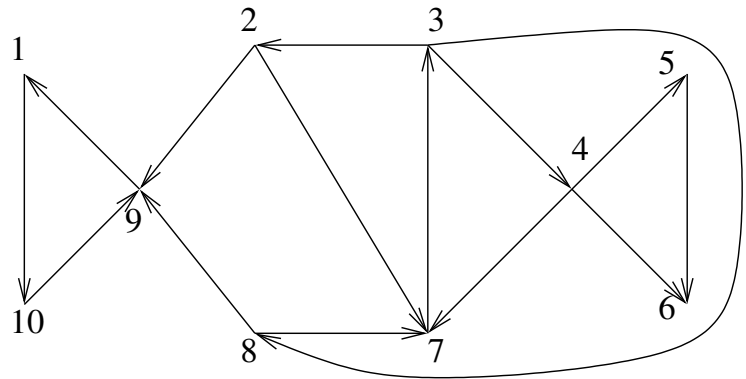
(a, 3pts) Please draw a picture of D .

(b, 3pts) Please write down the adjacency matrix M for D under the vertex order $(1, 2, 3, 4, 5, 6)$.

(c, 4pts) Please write down M^2 .

(d, 4pts) Please write down the reachability matrix R for D under the vertex order $(1, 2, 3, 4, 5, 6)$.

Problem 3 (14 points):
 Consider the following digraph:



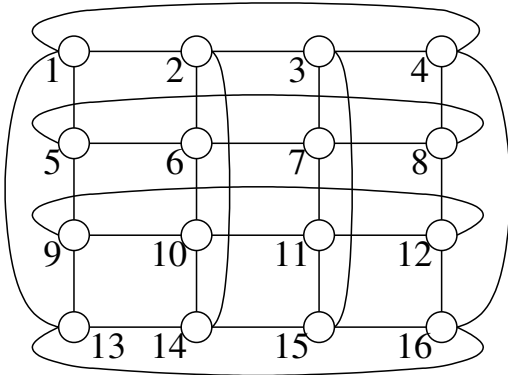
(a, 4pts) Please list the strong components of the above digraph.

(b, 4pts) Please draw the digraph that is the condensation of the above digraph.

(c, 4pts) Please list a vertex basis for the condensation and from it derive a vertex basis for the above digraph.

(d, 2pts) How many vertex bases does the above digraph have?

Problem 4 (10 points): Consider the 4×4 toric mesh graph with the vertices labeled 1 – 16 according to the figure below.



Please list all distinct shortest paths (called “chains” by the textbook for graphs) as sequences of vertices from vertex 1 to vertex 11.